How to Interpret Regression Coefficients and Calculate Adjustments for Differences in Property Productivity Features

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Abstract

In this article, a case study is presented to demonstrate use of a multiple regression analysis technique in the sales comparison approach to predict the market value of a commercial lot. In addition, the estimated parameters of the multiple regression model are used to calculate market-supported adjustments for the traditional direct sales comparison technique of the sales comparison approach. The multiple regression estimate has the predicted variable as the natural log of selling price. The predictor variables are a combination of natural log-transformed variable(s), variable(s) in original units, and specification of nominal and ordinal data as dummy variables.

Introduction

The sales comparison, income capitalization, and cost approaches may be applied by appraisers to develop a well-supported opinion of value. The sales comparison approached is described as,

> The process of deriving a value indication for the subject property by comparing similar properties that have recently sold with the property being appraised, identifying appropriate units of comparison, and making adjustments to the sale prices (or unit prices, as appropriate) of the comparable properties based on relevant, market-derived elements of comparison.[1]

Lusht, in his textbook, *Real Estate Valuation: Principles and Applications*, identifies three techniques of the sales comparison approach—direct sales comparison, regression analysis, and statistical analysis.[2] This article develops a case study applying regression analysis to the valuation of a commercial lot. In addition, the discussion shows the development of market-supported adjustments for the traditional direct sales comparison technique for valuation of the same property, based on the regression coefficients of the estimated regression model.

Supporting Adjustments in Direct Sales Comparison Technique of Sales Comparison Approach

Regulatory institutions and clients require support for analyses and conclusions rendered by appraisers. In the Uniform Standards of Professional Appraisal Practice (USPAP), the Comment component of the definition of *credible* states, "Credible assignment results require support, by relevant evidence and logic, to the

1. Appraisal Institute, *The Appraisal of Real Estate*, 14th ed. (Chicago: Appraisal Institute, 2013), 377.
2. Kenneth M. Lusht, *Real Estate Valuation: Principles and Applications* (State College, PA: KML Publishing, 2001), 119–138.

degree necessary for the intended use."[3]

The Uniform Appraisal Standards for Federal Land Acquisitions include the following statements on the need for support for adjustments in the appraisal process:[4]

- *Quantitative adjustments* should be made whenever adequate market data exist to support dollar or percentage amount adjustments. (Sec. 1.5.2.3)
- As with the sales comparison approach, the appraiser must collect market data to support adjustments (quantitative and/or qualitative) to the comparable leases for differences between them and the subject property. (Sec. 1.5.4.2)
- The level of market support for the adjustment process and the number and size of the adjustments should also be considered. (Sec. 1.6)
- The adjustments must be summarized in an adjustment grid and each adjustment (whether qualitative or quantitative) should be supported with market data. The data and analysis must provide sufficient detail for the client and intended users to understand the data, the analysis, and the logic of the appraiser's opinion of market value for the subject land as if vacant. (Sec 2.3.3.2.1)
- [Q]uantitative adjustment is not appropriate for characteristics for which reliable numerical adjustment cannot be derived from market data. (Sec. 4.4.2.2)

Fannie Mae's *Selling Guide* specifies that unacceptable appraisal practices include the following:

- Use of adjustments to comparable sales that do not reflect market reaction to the differences between the subject property and the comparable sales.
- Not supporting adjustments in the sales comparison approach.[5]

The *Selling Guide* also states,

> The appraiser's adjustments must reflect the market's reaction (that is, market-based adjustments) to the difference in the properties. For example, it would be inappropriate for an appraiser to provide a $20 per square foot adjustment for the difference in the gross living area based on a rule-of-thumb when market analysis indicates the adjustment should be $100 per square foot.[6]

The Appraisal of Real Estate, fourteenth edition, lists four groups of quantitative techniques to quantify adjustments for differences in properties, namely,[7]

1. Data analysis techniques—paired data analysis, grouped data analysis, and secondary data analysis
2. Statistical analysis—graphic analysis and scenario analysis
3. Cost-related adjustments—cost to cure, depreciated cost
4. Capitalization of income differences

That text, however, includes the following caution on the use of paired data analysis:

> Although paired data analysis of sales or rents is a theoretically sound method, it may be impractical and produce unreliable results when only a narrow sampling of sufficiently similar properties is available. This is particularly true for commercial and industrial properties that do not sell or lease frequently in the market. A lack of data can make quantifying the adjustments attributable to all the variables a difficult process. An adjustment derived from a single pair of sales is not necessarily indicative, just as a single sale does not necessarily reflect market value.[8]

In the easement condemnation case of *Guardian Pipeline, L.L.C. v. 950.80 Acres of Land, et al.*, Chief Judge Easterbrook, US Court of Appeals for the Seventh Circuit, suggests multiple regression as an alternative to matched pairs in the sales comparison approach. Easterbrook states as follows:

3. Appraisal Standards Board, *Uniform Standards of Professional Appraisal Practice*, 2018–2019 ed. (Washington DC: The Appraisal Foundation, 2018), Lines 112–113.
4. Interagency Land Acquisition Conference, "Adjustment Process," section 1.5.2.3 in *Uniform Appraisal Standards for Federal Land Acquisitions*, 2016 ed., http://bit.ly/UASFLA.
5. Fannie Mae, "Unacceptable Practices," Sec. B4-1.1-04 in *Selling Guide: Fannie Mae Single Family*, February 27, 2018, http://bit.ly/FannieMaeGuide.
6. Fannie Mae, "Adjustments to Comparable Sales," Sec. B4-1.3-09 in *Selling Guide*.
7. Appraisal Institute, *The Appraisal of Real Estate*, 14th ed., 398.
8. Appraisal Institute, *The Appraisal of Real Estate*, 14th ed., 399.

What puzzles us is why both sides were fixated on pair-wise comparisons—that is, have a transmission-corridor easement … appraising that parcel, and then comparing the appraised value of the "matched" parcel with the appraised values of the subject parcel with a pipeline easement. That process is full of problems. No other parcel will be identical to the subject parcel except for its lack of a transmission-corridor easement. Location and other attributes always differ, setting the stage for debate about whether an appropriate comparison has been selected. And even if very similar parcels can be found for comparison, the appraisals are just estimates. Each of these comparisons requires two appraisals: one of the "matched" parcel, and one (informed by the comparison) of the subject parcel with the easement.

A different approach would be to gather data about the actual selling prices of real estate with and without transmission-corridor easements and use these data to determine how much the easement reduces the value of real estate in real transactions. The law of large numbers would make up for the lack of closely matched comparison pairs. How many feet of transmission easement encumbers a parcel is a continuous variable and could be one independent variable in a regression. Daniel L. Rubinfeld, *Reference Guide on Multiple Regression*, in *Reference Manual on Scientific Evidence* 179–227 (Federal Judicial Center 2d ed. 2000), provides a good description. Using real transaction prices reduces the role of guesswork. Although no one suggested such an approach in this proceeding, litigants (and district judges) should keep it in mind for the future, as it has the potential to be faster, less expensive, and more accurate than a parade of witnesses offering estimates that cannot be verified.[9]

Previous Research

Numerous studies on the application of regression analysis in valuation analysis have been published in the appraisal literature, but few studies on using regression analysis to calculate adjustments for the direct sales comparison technique of the sales comparison approach. In 1989, Cannaday published an article on market support for adjustments for single-family home appraisals. The dependent variable in his study is the natural log of the sale price, and log-transformed independent variables are living area, bathrooms, car garage, lot area; months since sale and age are in their regular units, and corner and cul-de-sac locations are specified as dummy variables.[10]

Ramsland and Markham, in their *Appraisal Journal* article, demonstrate how to compute adjustments based on multiple regression analysis of warehouse/manufacturing properties. The dependent variable (sale price) and independent variables—elapsed time, gross building area, age at date of sale and location rating—are all in their original units in the estimated equation.[11]

Specification of the Regression Model

According to the fourteenth edition of *The Appraisal of Real Estate*, "The principle of contribution states that the value of a particular component is measured in terms of its contribution to the value of the whole property or as the amount that its absence would detract from the value of the whole."[12] *Contributory value* is defined in *The Dictionary of Real Estate Appraisal*, sixth edition, as "The change in value of a property as a whole, whether positive or negative, resulting from the addition or deletion of a property component."[13] The contributory value of a property component is explained by the economic concept of diminishing utility. *Diminishing utility* is defined in *The Dictionary of Real Estate Appraisal* as, "In economics, the concept that the consumption of each succeeding unit of an economic good yields less satisfaction than the preceding unit although satisfaction continues to increase at a positive rate. Thus, total utility increases at a decreasing rate."[14]

Depending on the nature of the nonlinear relationship, a curvilinear function captures the nonlinear relationship between selling price and some of the productivity features of a property.

9. As quoted in Peter F. Colwell, John A. Heller, and Joseph W. Trefzger, "Expert Testimony: Regression Analysis and Other Systematic Methodologies," *The Appraisal Journal* (Summer 2009): 253–262, 254.

10. Roger E. Cannaday, "How Should You Estimate and Provide Market Support for Adjustments in Single Family Appraisals?" *The Real Estate Appraisal & Analyst* (Winter 1989): 43–54.

11. Maxwell O. Ramsland, Jr., and Daniel E. Markham, "Market-Supported Adjustments Using Multiple Regression Analysis," *The Appraisal Journal* (April 1998): 181–191.

12. Appraisal Institute, *The Appraisal of Real Estate*, 14th ed., 32.

13. Appraisal Institute, *The Dictionary of Real Estate Appraisal*, 6th ed. (Chicago: Appraisal Institute, 2015), s.v. "contributory value."

14. Appraisal Institute, *The Dictionary of Real Estate Appraisal*, 6th ed., s.v. "diminishing utility."

For example, the relationship between selling price and building area often is not constant or a linear function but a curvilinear function that shows a diminishing marginal contribution to selling price as the building area increases. A curvilinear function represents a situation in which the impact of the predictor variable on the predicted variable is expected to increase at a decreasing rate as the predictor variable increases in magnitude, if the value of the coefficient of the predictor variable is between zero and one. For example, empirical evidence supports the observation that increases in selling price decrease as the size of the building increases.

Bryan and Colwell discuss several of the advantages of a non-linear model over a linear model. First, using the log of selling price as the dependent variable allows price increases to affect properties of different qualities in terms of percentage increases rather than dollar increases. Second, a non-linear model better handles the interdependencies or interactions among the predictor variables. For example, the concept of interdependency or interaction relates to the empirical evidence that the number of bathrooms in a home affects home price depending on the size of the home. Third, a non-linear model better handles non-linear partial effects—determining the impact of a predictor variable while holding other predictor variables in the model constant.[15]

The model specified for this study is the natural log of the dependent variable, natural logs of continuous variables, discrete variables in their original metrics or units, and dummy variables for qualitative variables. Using the natural log of a predictor variable is justified when its distribution is positively skewed (the mean is higher than the median) and the variability of the residuals for a dependent variable increase for larger values of the dependent variable. When a variable is log transformed, taking the antilog of the estimated coefficient will not properly transform the variable back into the original unit used. The correct way to transform a variable back to its original unit is presented in a later section.

Case Study: Regression Analysis Technique in Sales Comparison Approach

Case Study Data

Data for this case study are gathered on commercial-zoned lots in Council Bluffs, Iowa. The Missouri River serves as the Iowa/Nebraska state border, with Council Bluffs to the east of the river and Omaha, Nebraska, to the west. As of July 7, 2016, the estimated populations of Omaha and Council Bluffs were 446,697 and 62,524, respectively. Data are gathered on both sold properties and properties offered for sale (listings) on the following significant property productivity characteristics: selling price, sale date, site size, shape of site, corner site identification, and location. There are no significant differences in macro-level and micro-level accessibility characteristics. Exhibit 1 shows a summary of the data set and Exhibit 2 shows the definitions of variables.

In order to predict the expected selling price of a commercial lot, the following model is specified.

$$\ln(SP) = \beta_0 + \beta_1 \ln(LandSF) + \beta_2 SaleAge + \beta_3 ListingD_1 + \beta_4 CornerD_2 + \beta_5 ShapeD_3 + \beta_6 LocRating1D_4 + \beta_7 LocRating3D_5 + \varepsilon \quad [1]$$

The estimated model follows:

$$\ln(\hat{SP}) = \hat{\beta}_0 + \hat{\beta}_1 \ln(LandSF) + \hat{\beta}_2 SaleAge + \hat{\beta}_3 ListingD_1 + \hat{\beta}_4 CornerD_2 + \hat{\beta}_5 ShapeD_3 + \hat{\beta}_6 LocRating1D_4 + \hat{\beta}_7 LocRating3D_5 \quad [2]$$

15. Thomas B. Bryan and Peter F. Colwell, "Housing Price Indices," in vol. 2 of *Research in Real Estate*, ed. C. F. Sirmans (Greenwich, CT: JAI Press, Inc. 1982), 57–84.

Exhibit 1 Summary of Data Set and Coding

Sale/Listing	Selling/Listing Price ($)	DateSold	SaleAge	LandSF	Listing	ListingYes	Corner	CornerYes	Shape	ShapeRegYes	LocRating	LocRating1Yes	LocRating3Yes	LnLandSF	LnSellingPrice ($)
S1	885,000	4/30/11	40,663	74,052	No	0	Yes	1	Regular	1	2	0	0	11.21252283	13.6933429
S2	321,544	5/2/12	41,031	38,768	No	0	No	0	Regular	1	2	0	0	10.56535044	12.6808897
S3	464,000	9/3/09	40,059	41,817	No	0	No	0	Regular	1	2	0	0	10.64105823	13.0476398
S4	1,010,000	5/23/11	40,686	59,459	No	0	Yes	1	Regular	1	2	0	0	10.99304228	13.8254609
S5	485,000	7/19/11	40,743	35,283	No	0	No	0	Regular	1	3	0	1	10.47115654	13.0919042
S6	424,850	1/14/09	39,827	29,185	No	0	No	0	Regular	1	3	0	1	10.28141016	12.9594914
S7	695,000	4/20/12	41,019	50,529	No	0	No	0	Regular	1	3	0	1	10.83030271	13.4516671
S8	817,500	7/13/10	40,372	60,548	No	0	No	0	Regular	1	3	0	1	11.01119172	13.6140062
S9	865,000	11/12/10	40,494	67,082	No	0	No	0	Regular	1	3	0	1	11.11367103	13.6704848
S10	469,500	12/31/09	40,178	34,848	No	0	No	0	Regular	1	3	0	1	10.45875103	13.0594236
S11	1,184,832	9/28/07	39,353	69,696	No	0	No	0	Regular	1	3	0	1	11.15189821	13.9851116
S12	950,000	4/22/10	40,290	73,180	No	0	No	0	Regular	1	2	0	0	11.20067744	13.7642173
S13	856,000	1/16/06	38,733	71,308	No	0	Yes	1	Regular	1	2	0	0	11.1747638	13.6600257
S14	600,000	1/25/08	39,472	63,340	No	0	Yes	1	Regular	1	2	0	0	11.05627232	13.3046849
S15	1,600,000	1/4/08	39,451	156,031	No	0	No	0	Regular	1	2	0	0	11.95780998	14.2855142
S16	700,000	10/26/07	39,381	92,739	No	0	No	0	Regular	1	2	0	0	11.43754438	13.4588356
S17	476,534	3/15/07	39,156	90,082	No	0	No	0	Regular	1	2	0	0	11.40847565	13.0742944
S18	925,000	11/10/14	41,953	206,039	No	0	No	0	Regular	1	1	1	0	12.23581978	13.7375490
S19	468,380	10/21/16	42,664	81,457	No	0	Yes	1	Irregular	0	2	0	0	11.30783301	13.0570352
S20	1,300,000	4/21/17	42,846	203,861	No	0	No	0	Regular	1	2	0	0	12.22519269	14.0778748
L1	535,785	6/14/17	42,900	71,438	Yes	1	No	0	Regular	1	2	0	0	11.17658522	13.1914882
L2	490,050	6/14/17	42,900	65,340	Yes	1	No	0	Regular	1	2	0	0	11.08735969	13.1022627
L3	1,260,079	6/14/17	42,900	173,804	Yes	1	Yes	1	Regular	1	2	0	0	12.06568581	14.0466850
L4	2,799,166	6/14/17	42,900	311,018	Yes	1	Yes	1	Regular	1	2	0	0	12.64760735	14.8448319
L5	3,064,228	6/14/17	42,900	557,132	Yes	1	Yes	1	Irregular	0	2	0	0	13.23055819	14.9353063
L6	2,803,300	6/14/17	42,900	560,617	Yes	1	Yes	1	Irregular	0	2	0	0	13.2367936	14.8463079
L7	511,852	6/14/17	42,900	68,389	Yes	1	No	0	Regular	1	2	0	0	11.1329702	13.1457908
L8	475,792	6/14/17	42,900	63,598	Yes	1	Yes	1	Regular	1	2	0	0	11.06033101	13.0727361
L9	705,795	6/14/17	42,900	94,090	Yes	1	No	0	Irregular	0	2	0	0	11.4520028	13.4670801
L10	799,000	6/14/17	42,900	155,074	Yes	1	Yes	1	Irregular	0	1	1	0	11.95165512	13.5911162
L11	962,676	6/14/17	42,900	74,052	Yes	1	No	0	Regular	1	2	0	0	11.21252283	13.7774722
L12	789,000	6/14/17	42,900	131,551	Yes	1	No	0	Irregular	0	1	1	0	11.78715141	13.5785216
L13	650,000	6/14/17	42,900	155,945	Yes	1	No	0	Irregular	0	1	1	0	11.95725738	13.3847276
L14	350,000	6/14/17	42,900	54,886	Yes	1	No	0	Regular	1	1	1	0	10.9130063	12.7656884
L15	150,000	6/14/17	42,900	19,602	Yes	1	No	0	Regular	1	1	1	0	9.883386881	11.9183906
L16	500,000	6/14/17	42,900	28,750	Yes	0	Yes	1	Regular	1	3	0	1	10.26637913	13.1223634
SuPr		6/14/17	42,900	87,120	No	1	Yes	1	Regular	1	2	0	0	11.37504176	

Exhibit 2 Legend for Variable Definitions

ln(*SP*)	Natural logarithm of selling price
ln(*LandSF*)	Natural logarithm of size of lot in square feet
SaleAge	Time elapsed between sale date and effective date of appraisal
$ListingD_1$	Listing dummy variable = 1, if a listing, 0 otherwise
$CornerD_2$	Corner dummy variable = 1, if a corner lot, 0 otherwise
$ShapeD_3$	Shape of lot = 1, if regular, 0 otherwise
$LocRating1D_4$	D_4 = 1, if rating is 1, 0 otherwise
LocRating2	The base rating to which other location ratings are compared
$LocRating3D_5$	D_5 = 1, if rating is 3, 0 otherwise

Interpretations of Regression Dependent Variable and Coefficients

Ln($\hat{SP}$). The expected natural log of selling price of a lot is $\ln(\hat{SP})$. When the dependent variable is log transformed, to make a prediction in the original form (unlogged dependent variable), simply taking the exponent or antilog of the estimated model has been shown to systematically underestimate the expected dependent variable. To correct this underestimation of the dependent variable, a required correction to the estimated model is to add one-half of the square of the standard error of the regression model to the estimated model before taking the exponent or antilog. For example, if $\ln(SP) = \beta_0 + \beta_1 \ln(LandSF) + \varepsilon$ is the model estimated, prediction of selling price is made with $\hat{SP} = \exp(\hat{\beta}_0 + \hat{\beta}_1 \ln(LandSF) + se^2/2)$.[16] ($se^2$ is the square of the standard error of the regression or estimate.) In this study, to make a prediction of the expected selling price, the following equation is used:

$$\hat{SP} = \exp(\hat{\beta}_0 + \hat{\beta}_1 \ln(LandSF) + \hat{\beta}_2 SaleAge + \hat{\beta}_3 ListingD_1 + \hat{\beta}_4 CornerD_2 + \hat{\beta}_5 ShapeD_3 + \hat{\beta}_6 LocRating1D_4 + \hat{\beta}_7 LocRating3D_5 + se^2/2)$$

$\hat{\beta}_0$. $\hat{\beta}_0$ is the intercept of the regression line. It is the expected value of the dependent variable when all the predictor variables and the error term equal zero. Providing an economic interpretation to the intercept term is not always possible.[17] In their textbook, Johnson, Jr., Johnson, and Buse state, "because there are relatively few instances in applied research where the intercept has an obvious econometric interpretation, major concern is placed, here and elsewhere, on the slope coefficient, which does have an economic interpretation."[18] Cassidy in his textbook states,

> The constant term should not be relied on for inference because:
>
> 1. It absorbs the mean effects of all omitted variables, acting as a garbage term.
> 2. It absorbs any nonzero mean of the sample values of the error term, an additional garbage collector role.
> 3. It usually lies outside the relevant range of observations on the regressors, making inferences tenuous.
> 4. There may be nonlinearities in the true function in the range in which the regressors equal zero.[19]

16. Jeffrey M. Wooldridge, *Introductory Econometrics: A Modern Approach*, 6th ed. (Boston, MA: Cengage Learning, 2016), 190–191. Sanjiv Jaggia and Alison Kelly, *Business Statistics, Communicating with Numbers* (New Delhi, India: McGraw Hill Education (India) Private Limited, 2013), 500–501.
17. Jaggia and Kelly, *Business Statistics*, 433.
18. Aaron C. Johnson, Jr., Marvin B. Johnson, and Rueben C. Buse, *Econometrics, Basic and Applied* (New York: MacMillan Publishing Company, 1987), 54.
19. Henry J. Cassidy, *Using Econometrics: A Beginner's Guide* (Reston, Virginia: Reston Publishing Company, Inc., A Prentice Hall Company, 1981), 148.

$\hat{\beta}_1$ln(LandSF). Given the percentage change in ln*LandSF* (%Δln*LandSF*), holding other variables in the model constant, calculation of the unlogged percent change in the predicted selling price is made, with the following equation:[20]

$$\%\Delta(S\hat{P}) = [\exp(\hat{\beta}_1 * \ln(1+\%\Delta LandSF) - 1]*100 \quad [3]$$

When the dependent variable and a predictor variable are log transformed, the coefficient of the predictor variable is interpreted as elasticity. The coefficient measures the percentage change in the dependent variable for a given percentage change in the predictor variable.

$\hat{\beta}_2$SaleAge. *SaleAge* is the number of days that elapsed between the date a property sold and the effective date of the appraisal. The impact of $\hat{\beta}_2$ on the selling price is calculated using the equation that follows:[21]

$$\%\Delta(S\hat{P}) = [\exp(\hat{\beta}_2 * \Delta SaleAge) - 1]*100 \quad [4]$$

When the coefficient is small, Equation 4 can be approximated by $\hat{\beta}_2 * \Delta SaleAge * 100$. Because the dependent variable is log transformed and the predictor variable is not, the coefficient $\hat{\beta}_2$, may be interpreted as semi-elasticity. It measures the percentage change in the selling price resulting from one-unit change in *SaleAge*.

$\hat{\beta}_3$ListingD$_1$. Dummy variable *ListingD$_1$* represents a listing (offer to sell), a qualitative variable. If an observation is a listing, it is coded as 1 or as 0 otherwise. The percent adjustment to the selling price, for a listing, is calculated using one of the formulas presented below.

If the dummy variable switches from 0 to 1, the percentage impact of the dummy variable on expected selling price is

$$[\exp(\hat{\beta}_3 - se^2/2 \text{ for } \hat{\beta}_3) - 1]*100 \quad [5]$$

If the dummy variable switches from 1 to 0, the percentage impact of the dummy variable on expected selling price is

$$[\exp(-\hat{\beta}_3 - se^2/2 \text{ for } \hat{\beta}_3) - 1]*100 \quad [6]$$

where:

$\hat{\beta}_3$ is the estimated coefficient for listing dummy variable.[22]

$\hat{\beta}_4$CornerD$_2$. The dummy variable *CornerD$_2$* represents corner lot attribute, a qualitative variable. If an observation is a corner lot, it is coded as 1, or as 0 otherwise. The percent adjustment to the selling price for corner lot attribute is calculated similarly as was done for the listing variable above.

$\hat{\beta}_5$ShapeD$_3$. The dummy variable *ShapeD$_3$* represents shape of land attribute, a qualitative variable. If land is regular in shape, it is coded 1, or as 0 if shape is irregular. It is interpreted similarly as was done for D_1 above, the dummy variable for the listing variable.

Location (*$\hat{\beta}_6$LocRating1D$_4$* and *$\hat{\beta}_7$LocRating3D$_5$*). *LocRating1D$_4$* and *LocRating3D$_5$* represent ratings of locations specified as dummy variables for data-processing purposes. Based on the principle of land use association and economics of agglomeration, the locations of the lots are rated from a

20. Jing Yang, "Interpreting Coefficients in Regression with Log-Transformed Variables," *StatNews #83* (Cornell University: Cornell University Consulting Unit, June 2012), http://bit.ly/stnews83; Kenneth Benoit, "Linear Regression Models with Logarithmic Transformations" (Methodology Institute, London School of Economics, March 17, 2011), http://bit.ly/BenoitModels.

21. Yang, "Interpreting Coefficients in Regression"; Benoit, "Linear Regression Models"; Wooldridge, *Introductory Econometrics*, 6th ed., 171–173;

22. Peter E. Kennedy, "Estimation with Correctly Interpreted Dummy Variables in Semilogarithmic Equations," *The American Economic Review* 71, no. 4 (September 1981): 801; *Econometrics Beat: Dave Giles' Blog*, "Dummies for Dummies," posted March 24, 2011, http://bit.ly/Gilesblog.

low of 1 to a high of 3. Location is an ordinal variable and the rating of location on a scale of 1 to 3, represents an ordinal scale of measurement. Differences between rating values are meaningless because the actual numbers used are arbitrary. However, a location rated a 3 is better than a location rated a 2, and a location rated a 2 is better than a location rated 1. If location is included in the estimated equation as any other discrete or continuous variable, then its coefficient would be the percentage point change in selling price when location rating increases by one unit, holding other predictor variables constant. This would imply that a one-unit increase in location rating has a constant impact on selling price. For the above reasons, the ordinal location variable is specified as a dummy variable. Since ratings 1, 2, and 3 are used, two dummy variables are used to specify location ratings.[23] The percent adjustments to the selling price for location attribute are calculated similarly as was done for the listing variable above.

The regression model is estimated using NCSS Statistical Software and the regression report is presented as Exhibit 3.[24]

Predicted Expected Selling Price

The unadjusted coefficient of determination, *R*-squared, is 0.8897, and the adjusted *R*-squared is 0.8621. The adjusted *R*-squared value is interpreted to mean that 86.21% of the variation in the natural log of selling price is explained by the estimated model. The *F*-test is used to test whether the estimated model is statistically significant. The *F*-ratio, from the Analysis of Variance section in Exhibit 3, is 32.265 and the *p*-value is 0.0000. The *p*-value of 0.0000 indicates the probability of observing an *F*-ratio at least as large as 32.265, if the estimated model is not statistically significant. All the coefficients of the estimated model have the expected signs, and they range in *p*-values of 0.0000 to 0.6023 for the *t*-statistic. A small *p*-value is required to reject the null hypothesis that an estimated coefficient is equal to zero.

In a valuation context, regression analysis is used to predict the expected selling price or expected rental price. The main focus should be economic or practical significance and not statistical significance. The statistical significance of a predictor variable depends solely on the magnitude of the *t*-statistic for the variable. The economic or practical significance of a predictor variable depends on the size and sign of the coefficient of the variable.[25] The smaller the sample size, the harder to find a variable to be statistically significant. Wooldridge states as follows:

> Many researchers are also willing to entertain larger significance levels in applications with small sample sizes, reflecting the fact that it is harder to find significance with smaller sample sizes. … [O]ne's willingness to consider higher significance levels can depend on one's underlying agenda.

Given that in valuation the focus is prediction, it is the economic or practical significance of a predictor variable that should be the emphasis, as long as the coefficient has the expected sign. Typically, one has a limited number of comparable properties available for analysis. Therefore, a very high level of statistical significance would be reasonable for a variable if its coefficient has the expected sign.

The estimated equation from Exhibit 3 is repeated as follows:

$$\begin{aligned}&\ln SellingPrice = \\ &5.98147597210732 + 0.155857423013752 \\ &* (CornerYes=1) + 0.0719102539596642 \\ &* (ListingYes=1) + 0.831544302361704 \\ &* \ln LandSF - 0.282929997857729 \\ &* (LocRating1Yes=1) + 0.398284134042261 \\ &* (LocRating3Yes=1) - 5.27680229331659E\text{-}05 \\ &* SaleAge + 0.217073232411639 \\ &* (ShapeRegYes=1) \qquad [7]\end{aligned}$$

To convert back to its original unit, simply finding its antilog would underestimate the expected selling price. The estimated equation is adjusted by adding one half of the square of the standard error of the estimated equation to it before finding the antilog of the modified equation as shown below.[26]

23. Wooldridge, *Introductory Econometrics*, 6th ed., 214–215.

24. NCSS, LLC, NCSS Statistical Software, Version 10 (Kaysville, UT).

25. Wooldridge, *Introductory Econometrics*, 6th ed., 120–121.

26. Jaggia and Kelly, *Business Statistics*, 501.

Exhibit 3 Multiple Regression Report

Run Summary Report

Item	Value	Rows	Value
Dependent Variable	ln*SellingPrice*	Rows Processed	37
Number Ind. Variables	7	Rows Filtered Out	0
Weight Variable	None	Rows with *X* Missing	0
R^2	0.8897	Rows with Weight Missing	0
Adj. R^2	0.8621	Rows with *Y* Missing	1
Coefficient of Variation	0.0169	Rows Used in Estimation	36
Mean Square Error	0.05239125	Sum of Weights	36.000
Sq. Root MSE	0.2288913		
Ave Abs Pct Error	1.130		
Completion Status	Normal Completion		

Descriptive Statistics

Variable	Count	Mean	Standard Deviation	Minimum	Maximum
(*CornerYes*=1)	36	0.3333333	0.4780914	0	1
(*ListingYes*=1)	36	0.4166667	0.5	0	1
Ln*LandSF*	36	11.32767	0.7652684	9.883387	13.23679
(*LocRating1Yes*=1)	36	0.1666667	0.3779645	0	1
(*LocRating3Yes*=1)	36	0.2222222	0.421637	0	1
SaleAge	36	41521.42	1493.185	38733	42900
(*ShapeRegYes*=1)	36	0.8055556	0.4013865	0	1
Ln*SellingPrice*	36	13.50806	0.6164367	11.91839	14.93531

Regression Coefficients *T*-Tests

Independent Variable	Regression Coefficient b(i)	Standard Error Sb(i)	Standardized Coefficient	*T*-Statistic to Test H0: β(i)=0	Prob Level	Reject H0 at 10%?	Power of Test at 10%
Intercept	5.981476	1.961028	0.0000	3.050	0.0050	Yes	0.9083
(*CornerYes*=1)	0.1558574	0.09106075	0.1209	1.712	0.0980	Yes	0.5105
(*ListingYes*=1)	0.07191025	0.1364376	0.0583	0.527	0.6023	No	0.1446
Ln*LandSF*	0.8315443	0.0664111	1.0323	12.521	0.0000	Yes	1.0000
(*LocRating1Yes*=1)	-0.28293	0.1219704	-0.1735	-2.320	0.0279	Yes	0.7318
(*LocRating3Yes*=1)	0.3982841	0.1127642	0.2724	3.532	0.0015	Yes	0.9641
SaleAge	-5.276802E-05	4.383917E-05	-0.1278	-1.204	0.2388	No	0.3215
(*ShapeRegYes*=1)	0.2170732	0.1277324	0.1413	1.699	0.1003	No	0.5058

Estimated Equation

ln*SellingPrice* =
5.98147597210732 + 0.155857423013752 * (*CornerYes*=1) + 0.0719102539596642 * (*ListingYes*=1) +
0.831544302361704 * ln*LandSF* – 0.282929997857729 * (*LocRating1Yes*=1) + 0.398284134042261 *
(*LocRating3Yes*=1) – 5.27680229331659E-05 * *SaleAge* + 0.217073232411639 * (*ShapeRegYes*=1)

CONTINUED >

Exhibit 3 Multiple Regression Report (*continued*)

Analysis of Variance

Source	DF	R^2	Sum of Squares	Mean Square	*F*-Ratio	Prob Level	Power (10%)
Intercept	1		6568.838	6568.838			
Model	7	0.8897	11.83284	1.690406	32.265	0.0000	1.0000
Error	28	0.1103	1.466955	0.05239125			
Total (Adjusted)	35	1.0000	13.2998	0.3799942			

Predicted Values with Prediction Limits of Individuals

Row	Actual Ln*SellingPrice*	Predicted Ln*SellingPrice*	Standard Error of Predicted	75% Lower Pred. Limit of Individual	75% Upper Pred. Limit of Individual
1	13.69334	13.53241	0.2443587	13.24536	13.81946
2	12.68089	12.81898	0.246209	12.52975	13.10821
3	13.04764	12.93323	0.2437253	12.64692	13.21954
4	13.82546	13.34869	0.2456157	13.06016	13.63722
5	13.0919	13.15414	0.2435165	12.86807	13.4402
6	12.95949	13.04469	0.2466298	12.75497	13.33441
7	13.45167	13.43822	0.2440331	13.15155	13.72489
8	13.61401	13.62278	0.2442199	13.33589	13.90967
9	13.67048	13.70156	0.2448024	13.41398	13.98913
10	13.05942	13.17363	0.2440782	12.88691	13.46036
11	13.98511	13.79355	0.2508593	13.49886	14.08824
12	13.76422	13.38638	0.2403078	13.10409	13.66868
13	13.66003	13.60285	0.2580428	13.29973	13.90598
14	13.30468	13.46533	0.2502194	13.17139	13.75927
15	14.28551	14.06025	0.2481098	13.76879	14.35171
16	13.45884	13.63132	0.2442448	13.3444	13.91824
17	13.07429	13.61902	0.246011	13.33002	13.90801
18	13.73755	13.87647	0.2805982	13.54684	14.20609
19	13.05704	13.289	0.2779678	12.96247	13.61554
20	14.07788	14.10344	0.2726921	13.7831	14.42378
21	13.19149	13.30054	0.2453283	13.01234	13.58873
22	13.10226	13.22634	0.2456836	12.93773	13.51495
23	14.04669	14.19572	0.2503918	13.90158	14.48986
24	14.84483	14.67961	0.2585801	14.37585	14.98337
25	14.93531	14.94729	0.25813	14.64406	15.25052
26	14.84631	14.95248	0.2582219	14.64914	15.25581
27	13.14579	13.26427	0.2454842	12.97589	13.55264
28	13.07274	13.35972	0.2500677	13.06596	13.65348
29	13.46708	13.31249	0.2635932	13.00284	13.62213
30	13.59112	13.6009	0.2609096	13.2944	13.90739
31	13.77747	13.33042	0.2452256	13.04235	13.61849
32	13.57852	13.30825	0.2550138	13.00868	13.60782
33	13.38473	13.4497	0.2550577	13.15008	13.74932
34	12.76569	12.79843	0.2549729	12.49891	13.09795
35	11.91839	11.94226	0.2660129	11.62976	12.25475
36	13.12236	13.02589	0.2731223	12.70505	13.34673
37		13.62142	0.2482459	13.3298	13.91304

$$SellingPrice = \exp(5.98147597210732 + 0.155857423013752 * (CornerYes{=}1) + 0.0719102539596642 * (ListingYes{=}1) + 0.831544302361704 * \ln LandSF - 0.282929997857729 * (LocRating1Yes{=}1) + 0.398284134042261 * (LocRating3Yes{=}1) - 5.27680229331659E\text{-}05 * SaleAge + 0.217073232411639 * (ShapeRegYes{=}1) + se^2/2) \quad [8]$$

Note, the standard error of regression, *se*, is also called standard error of the estimate or root mean squared error. In NCSS Statistical Software, it is called Square Root of MSE (shown in the Summary Report section of Exhibit 3). In Exhibit 3, Row 37 of the Prediction Values with Prediction Limits of Individuals section is the subject site and this information is repeated below.

Row	Actual LnSellingPrice	Predicted LnSellingPrice	Standard Error of Predicted	95% Lower Pred. Limit of Individual	95% Upper Pred. Limit of Individual
37		13.62142	0.2482459	13.3298	13.91304

In this instance, the predicted selling price, in its original unit or unlogged, is calculated as follows:

$$Selling\ Price = \exp(13.62142 + (0.2288913^\wedge 2)/2) \quad [9]$$

$$= \exp(13.6476156)$$

$$= \$845{,}442.65$$
($845,000.00 rounded)

Conclusion. Applying regression analysis to the market data and giving consideration to functional utility, location, physical attributes, and trend in property values in the immediate market area, the expected selling price is concluded to be $845,000.00 (rounded) or $9.70 per square foot for the 87,120 square feet of the subject's site area.

Case Study: Direct Sales Comparison Technique of Sales Comparison Approach

In applying the direct sales comparison technique, a value indication is derived by comparing the property being valued to similar properties that recently sold or are currently offered for sale in the market area. The basic steps to develop an indication of value for the property being appraised, via the direct sales comparison technique, are as follows:

1. The market area in which the property being appraised competes with other properties is researched to gather data on comparable sales and listings.
2. Sales and listings that are most similar to the subject property are selected.
3. The most meaningful unit of value for the appraisal subject is selected.
4. Each selected sale or listing is compared to the subject property on the following characteristics or attributes and adjustments made for significant value-influencing differences: real property rights conveyed, financing terms, conditions of sale, expenditures made immediately after purchase, market conditions over time, location, physical characteristics, economic characteristics, use (zoning), and non-realty components of value.
5. The value indicated for the subject property by each comparable sale is examined with respect to data input and analysis. The value indications are used to conclude a range or a point value for the subject property.

Current appraisal standards require that quantitative adjustments should not be made to a sale, unless the adjustment can be supported. In this instance, supported adjustments to the comparable sales and listings for significant differences between them and the property appraised, are derived from the coefficients or multipliers of the estimated regression equation in the preceding regression analysis technique section.

Sales Data and Analysis Grid

The sales and listings used in the direct sales comparison are a subset of the data set used in the preceding regression analysis technique. They are more physically and geographically proximate to the property appraised. Seven closed sales (Sales 1, 4, 7, 12, 15, 16, and 20) and three active listings (Listings 1, 2, and 11) are researched and analyzed to develop an opinion of market value for the subject site. The sales range in closing date from 10/26/2007 to 4/21/2017. The sales range in size from 41,817 to 203,861 square feet, and the unadjusted unit prices range from $6.38 to $16.99 per square foot of site area. The listings range in size from 65,340 to 74,052 square feet and the unadjusted list prices range from $7.50 to $13.00 per square foot of site area.

The properties in the data set are significantly similar in real property rights conveyed, financing, conditions of sale, expenditures made immediately after purchase, economic characteristics, use/zoning, and non-realty components; therefore, no adjustments were made for these characteristics. Exhibit 4 shows the adjustments made for property differences, followed by explanations for the calculations of the adjustments. Exhibit 5 repeats the regression coefficients information from Exhibit 3.

Listing Adjustment. Typically, the price paid for real property is less than the asking price. Typically, the asking price is set at a higher level than its market value to establish a range to negotiate the transaction price. If a site is a listing, it is coded 1 and it is coded 0 if it is an actual sale. Although the listing variable has a *p*-value of 0.6023, it is economically significant because its coefficient has the expected sign. To convert a listing to a sale, Equation 6, repeated below, is used.

$$[\exp(-\hat{\beta}_3 - se^2/2 \text{ for } \hat{\beta}_3) - 1]*100$$

Using Equation 6, the negative adjustment to a listing price is -7.80%.

Market Conditions Adjustment. Adjustment for changes in market conditions account for inflation, deflation, and changes in supply and demand in the market for the property between the date a property sold and the effective date of appraisal of the property valued. If there are changes in market conditions, a comparable property would have sold for a different price as of the effective date of appraisal of the subject property and an adjustment would be necessary for significant changes in market conditions. The coefficient for *SaleAge* is negative, implying declining prices after the date of sale; therefore, sales would require downward adjustments for declining prices. The *p*-value for *SaleAge* is 0.2388 and it is economically significant. Equation 4, repeated below is used to calculate the market condition adjustments presented in Exhibit 6.

$$\%\Delta(\hat{SP}) = [\exp(\hat{\beta}_2 * \Delta SaleAge) - 1]*100$$

Location Adjustment. Location attribute is considered in terms of friction of movement between linked sites. Linkage refers to time and distance relationships between two land use activities that generate movement of people and goods between them. The ratings were from a low of 1 to a high of 3. The location for the subject property is rated 2, and it is considered the base level with respect to ratings 1 and 3. The signs of the coefficients are as expected and the *p*-values for ratings 1 and 3 are 0.0279 and 0.0015, respectively. Sale 7, rated 3, requires a downward adjusted that is calculated with Equation 6 which is repeated as follows:

$$[\exp(-\hat{\beta}_7 - se^2/2 \text{ for } \hat{\beta}_7) - 1]*100$$

The calculated percent adjustment to the market conditions adjusted price is -47.98%.

Land Size Adjustment. The coefficient for site size has a *p*-value of 0.0000. The dependent variable as well as the predictor variable are in natural log units. Therefore, the fitted coefficient is an elasticity measure and percentage adjustments are made to the dependent variable. The formula for calculating the adjustment for difference in site size is Equation 3 repeated below:

$$\%\Delta(\hat{SP}) = [\exp(\hat{\beta}_1 * \ln(1+\%\Delta LandSF) - 1]*100$$

The calculated adjustments are in column 9 of Exhibit 7.

Corner Adjustment. The percent adjustment for corner lot location is calculated using Equation 5 below.

$$[\exp(\hat{\beta}_4 - se^2/2 \text{ for } \hat{\beta}_4) - 1]*100$$

Using the above formula, the calculated adjustment is 16.38% to the market conditions adjusted prices.

Exhibit 4 Summary & Adjustment Grid

Element	Subject	Sale 1	Sale 4	Sale 7	Sale 12	Sale 15
Sale price ($)	N/A	885,000	464,000	1,010,000	600,000	1,600,000
Site area (sq. ft.)	87,120	74,052	41,817	59,459	63,340	156,031
Date sold	6/14/17	4/30/11	5/23/11	4/20/12	4/22/10	1/4/08
Location rating	LocRating 2	LocRating 2	LocRating 2	LocRating 3	LocRating 2	LocRating 2
Corner	Yes	Yes	Yes	No	No	No
Shape of lot	Regular	Regular	Regular	Regular	Regular	Regular
Real property rights conveyed adjustment ($)		11.95	11.10	16.99	9.47	10.25
Adjusted price ($)		885,000	464,000	1,010,000	600,000	1,600,000
Financing adjustment ($)		-	-		-	-
Adjusted price ($)		885,000	464,000	1,010,000	600,000	1,600,000
Conditions of sale adjustment ($)		-	-		-	-
Adjusted price ($)		885,000	464,000	1,010,000	600,000	1,600,000
Expenditures made immediately after purchase adjustment ($)		-	-	-	-	-
Adjusted price ($)		885,000	464,000	1,010,000	600,000	1,600,000
Listing adjustment ($)		-	-	-	-	-
Adjusted price ($)		885,000	464,000	1,010,000	600,000	1,600,000
Market conditions adjustment ($)	Current	(104,467)	(54,208)	(100,249)	(82,635)	(291,195)
Adjusted price ($)		780,533	409,792	909,751	517,365	1,308,805
Location rating adjustment ($)		-	-	(436,498)	-	-
Adjusted price ($)		780,533	409,792	473,252	517,365	1,308,805
Land SF adjustment ($)		112,943	344,676	340,156	157,020	(502,712)
Adjusted price ($)		893,476	754,467	813,408	674,386	806,093
Corner adjustment ($)		-	-	149,017	84,744	214,382
Adjusted price ($)		893,476	754,467	962,425	759,130	1,020,475
Shape of lot ($)		-	-	-	-	-
Economic characteristics adjustment ($)		-	-		-	-
Use/zoning adjustment ($)		-	-		-	-
Non-realty components of value adj. ($)		-	-		-	-
Final adjusted price ($)		893,476	754,467	962,425	759,130	1,020,475
Indicated Market Value for Subject ($/SF)		10.26	8.66	11.05	8.71	11.71

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Exhibit 4 Summary & Adjustment Grid (*continued*)

Element	Subject	Sale 16	Sale 20	Listing 1	Listing 2	Listing 11
Sale price ($)	N/A	700,000	1,300,000	535,785	490,050	962,676
Site area (sq. ft.)	87,120	92,739	203,861	71,438	65,340	74,052
Date sold	6/14/17	10/26/07	4/21/17	6/14/17	6/14/17	6/14/17
Location rating	LocRating 2	LocRating 2	LocRating 2	LocRating 2	LocRating 2	LocRating 2
Corner	Yes	No	No	No	No	No
Shape of lot	Regular	Regular	Regular	Regular	Regular	Regular
Real property rights conveyed adjustment ($)		7.55	6.38	7.50	7.50	13.00
Adjusted price ($)		700,000	1,300,000	535,785	490,050	962,676
Financing adjustment ($)		-	-	-	-	-
Adjusted price ($)		700,000	1,300,000	535,785	490,050	962,676
Conditions of sale adjustment ($)		-	-	-	-	-
Adjusted price ($)		700,000	1,300,000	535,785	490,050	962,676
Expenditures made immediately after purchase adjustment ($)		-	-	-	-	-
Adjusted price ($)		700,000	1,300,000	535,785	490,050	962,676
Listing adjustment ($)		-	-	(41,791)	(38,224)	(75,089)
Adjusted price ($)		700,000	1,300,000	493,994	451,826	887,587
Market conditions adjustment ($)	Current	(129,983)	(3,704)	-	-	-
Adjusted price ($)		570,017	1,296,296	493,994	451,826	887,587
Location rating adjustment ($)		-	-	-	-	-
Adjusted price ($)		570,017	1,296,296	493,994	451,826	887,587
Land SF adjustment ($)		(28,843)	(656,963)	88,622	122,129	128,434
Adjusted price ($)		541,174	639,333	582,616	573,955	1,016,021
Corner adjustment ($)		93,369	212,333	80,916	74,009	128,434
Adjusted price ($)		634,542	851,666	663,532	647,964	1,144,455
Shape of lot ($)		-	-	-	-	-
Economic characteristics adjustment ($)		-	-	-	-	
Use/zoning adjustment ($)		-	-	-	-	-
Non-realty components of value adj. ($)		-	-	-	-	-
Final adjusted price ($)		634,542	851,666	663,532	647,964	1,144,455
Indicated Market Value for Subject ($/SF)		7.28	9.78	7.62	7.44	13.14

Final Adjusted Unit Price	
No. of Comparables	10
Maximum ($)	13.14
Average	9.56
Median	9.24
Minimum	7.28
Standard Deviation ($)	1.98

Indicated Value for Property Appraised	
Land Area (SF)	87,120
Indicated Value per SF	9.60
Subject's Indicated Value ($)	836,352.00
Rounded to ($)	836,000

Exhibit 5 Regression Coefficients *T*-Tests

Independent Variable	Regression Coefficient b(i)	Standard Error Sb(i)	Standardized Coefficient	*T*-Statistic to Test H0: $\beta(i)=0$	Prob Level	Reject H0 at 10%?	Power of Test at 10%
Intercept	5.981476	1.961028	0.0000	3.050	0.0050	Yes	0.9083
(*CornerYes=1*)	0.1558574	0.09106075	0.1209	1.712	0.0980	Yes	0.5105
(*ListingYes=1*)	0.07191025	0.1364376	0.0583	0.527	0.6023	No	0.1446
Ln*LandSF*	0.8315443	0.0664111	1.0323	12.521	0.0000	Yes	1.0000
(*LocRating1Yes=1*)	-0.28293	0.1219704	-0.1735	-2.320	0.0279	Yes	0.7318
(*LocRating3Yes=1*)	0.3982841	0.1127642	0.2724	3.532	0.0015	Yes	0.9641
SaleAge	-5.276802E-05	4.383917E-05	-0.1278	-1.204	0.2388	No	0.3215
(*ShapeRegYes=1*)	0.2170732	0.1277324	0.1413	1.699	0.1003	No	0.5058

Exhibit 6 Market Conditions Adjustments

Col. 1	Col. 2	Col. 3	Col. 4	Col. 5	Col. 6	Col. 7 = Col. 2 × Col. 5 × Col. 6
Sale/Listing	Selling/Listing Price ($)	Date Sold	Appraisal Date	Difference in Days	SaleAge Coefficient	Market Conditions Adjustment ($)
Sale 1	885,000	4/30/11	6/14/17	2237	-5.28E-05	(104,467)
Sale 4	464,000	5/23/11	6/14/17	2214	-5.28E-05	(54,208)
Sale 7	1,010,000	4/20/12	6/14/17	1881	-5.28E-05	(100,249)
Sale 12	600,000	4/22/10	6/14/17	2610	-5.28E-05	(82,635)
Sale 15	1,600,000	1/4/08	6/14/17	3449	-5.28E-05	(291,195)
Sale 16	700,000	10/26/07	6/14/17	3519	-5.28E-05	(129,983)
Sale 20	1,300,000	4/21/17	6/14/17	54	-5.28E-05	(3,704)
Listing 1	535,785	6/14/17	6/14/17	0	-5.28E-05	-
Listing 2	490,050	6/14/17	6/14/17	0	-5.28E-05	-
Listing 11	962,676	6/14/17	6/14/17	0	-5.28E-05	-

Exhibit 7 Adjustments for Differences in Land Size

Col. 1	Col. 2	Col. 3	Col. 4	Col. 5	Col. 6	Col. 7	Col. 8	Col. 9
Sale/ Listing	Comparable's Site Size (SF)	Subject's Site Size (SF)	%Δ LandSF	1 +%Δ LandSF	ln(1 +%Δ LandSF)	Coeff. For LnLandSF	exp(Col. 6 × Col. 7)	% Adjustment (Col. 8 – 1) × 100
Sale 1	74,052	87,120	0.1765	1.1765	0.162518929	0.8315443	1.14469896464643	14.47
Sale 4	41,817	87,120	1.0834	2.0834	0.733983523	0.8315443	1.84105691128422	84.11
Sale 7	59,459	87,120	0.4652	1.4652	0.381999479	0.8315443	1.37389461240575	37.39
Sale 12	63,340	87,120	0.3754	1.3754	0.318769438	0.8315443	1.30352340375450	30.35
Sale 15	156,031	87,120	-0.4416	0.5584	-0.582768227	0.8315443	0.61594500486970	-38.41
Sale 16	92,739	87,120	-0.0606	0.9394	-0.062502617	0.8315443	0.94935383900808	-5.06
Sale 20	203,861	87,120	-0.5726	0.4274	-0.85015191	0.8315443	0.49315143818992	-50.68
Listing 1	71,438	87,120	0.2195	1.2195	0.198456538	0.8315443	1.17942307915153	17.94
Listing 2	65,340	87,120	0.3333	1.3333	0.287682072	0.8315443	1.27025845489821	27.03
Listing 11	74,052	87,120	0.1765	1.1765	0.162518929	0.8315443	1.14469896464643	14.47

Shape of Lot Adjustment. The subject and comparable properties are all regular in shape. It is hypothesized that the impact of shape of a site on selling price is dependent on its size. It is likely that the functional utility of a large irregularly shaped site may not be as negatively impacted compared to that of a small site. An interaction term would be created by multiplying size of lot by its shape to capture the interdependency of the two variables. In this analysis, the interaction term was dropped from the final estimated equation due to multicollinearity problems.

Conclusion. In concluding value, the sales and listings analyzed may be weighted differently based on geographic and physical proximities to the property appraised. In this instance, the final value is based on the average of the final adjusted prices. Considering the market data analyzed, the final concluded value via the direct sales comparison technique of the sales approach is $836,000 as of June 14, 2017.

Conclusion

In this study, two techniques of the sales comparison approach are applied—regression analysis and the traditional direct sales comparison. The regression analysis provides an estimate of market value of $845,000, while the estimate via the traditional direct sales comparison is $836,000, weighting equally the final adjusted prices of the ten properties analyzed. The regression analysis prediction is higher by $9,000 or 1.07%. The regression analysis technique is considered superior to the traditional direct sales comparison because (1) it uses more observations to predict expected market value, (2) it provides a confidence interval for the estimated mean selling price, and (3) it provides probability values associated with the expected selling price and its confidence interval.

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About the Author

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Additional Resources

Suggested by the Y. T. and Louise Lee Lum Library

American Statistical Association, Publications
http://www.amstat.org/asa/publications/home.aspx

Appraisal Institute

- **Education**
  http://www.appraisalinstitute.org/assets/1/7/aiedcat.pdf
  - *Advanced Spreadsheet Modeling for Valuation Applications*
  - *Application & Interpretation of Simple Linear Regression*
  - *Comparative Analysis*
  - *General Appraiser Sales Comparison Approach*
  - *Quantitative Analysis*
  - *Real Estate Finance, Statistics, and Valuation Modeling*

- **Publications**
  - *A Guide to Appraisal Valuation Modeling*
    https://www.appraisalinstitute.org/a-guide-to-appraisal-valuation-modeling/
  - *An Introduction to Statistics for Appraisers*
    https://www.appraisalinstitute.org/an-introduction-to-statistics-for-appraisers/
  - *Practical Applications in Appraisal Valuation Modeling*
    https://www.appraisalinstitute.org/practical-applications-in-appraisal-valuation-modeling/
  - *Valuation by Comparison: Residential Analysis and Logic*
    https://www.appraisalinstitute.org/valuation-by-comparison-residential-analysis-and-logic/

Cornell Statistical Consulting Unit—*StatNews* (statistical topics monographs)
https://www.cscu.cornell.edu/news/archive.php