Regression Analysis of Property Productivity Index and Value

by A. Ason Okoruwa, PhD, MAI, AI-GRS

Abstract

In this case study, regression analysis is applied to the property productivity indexes of properties (price-productivity index regression), in the valuation of a light industrial property. The property productivity features selected for analysis are features justified by economic theory in the empirical real estate literature. The weights assigned to these features are determined using the Solver program, a Microsoft Excel add-in. This approach introduces objectivity in evaluating the value-influencing features of a property and their weights. Excel is used for the one-predictor variable regression.

Statement of the Problem

A common objective of an appraisal is to develop an opinion of the market value of an interest in the property being appraised. To accomplish this objective, the appraiser has to research and analyze market data. The appraiser is supposed to replicate the price-establishing behavior of the relevant buyer group in the real estate market.

A prospective buyer will evaluate the productive attributes of different properties in deciding which property to buy. Given differences in the bundles of property productivity attributes of the properties being evaluated, a critical question is how do prospective buyers evaluate the productivity attributes of different comparable and competitive properties. The prospective buyers will commonly rate the productivity attributes of different properties in making a purchase decision.

The sales comparison approach is often employed by appraisers in developing an opinion of value of the interest in an owner-occupied industrial or commercial property. The *sales comparison approach* can be described as follows:

> The process of deriving a value indication for the subject property by comparing similar properties that have recently sold with the property being appraised, identifying appropriate units of comparison, and making adjustments to the sale prices (or unit prices, as appropriate) of the comparable properties based on relevant, market-derived elements of comparison.[1]

To develop an opinion of the market value of a property, comparative analysis is typically employed. *Comparative analysis* is "the process by which a value indication is derived in the sales comparison approach. Comparative analysis may employ quantitative or qualitative techniques, either separately or in combination."[2]

Given that no two properties are alike in their bundle of productivity attributes, an appraiser must adjust for differences in attributes. *The Appraisal of Real Estate*, fourteenth edition, provides a systematic procedure for adjustments in the sales comparison approach. It advises appraisers as follows:

> Look for differences between the comparable sale properties and the subject property using all appropriate elements of comparison. Then adjust the price of each sale property, reflecting how it differs, to equate it to the subject property or eliminate that property as a comparable. This step typically involves using the most similar sale properties and then adjusting for any remaining

1. Appraisal Institute, *The Appraisal of Real Estate*, 14th ed. (Chicago: Appraisal Institute, 2013), 377.
2. Ibid., 397.

> differences. If a transaction does not reflect the actions of a buyer who would also be attracted to the subject property, the appraiser should be concerned about comparability.[3]

The Appraisal of Real Estate, fourteenth edition, lists four broad groups of quantitative techniques used to quantify adjustments for differences, namely,[4]

1. Data analysis techniques, such as paired data analysis, grouped data analysis, and secondary data analysis
2. Statistical analysis, including graphic analysis and scenario analysis
3. Cost-related adjustments (cost to cure and depreciated cost)
4. Capitalization of income differences

Of these techniques, paired data analysis is the dominant quantitative technique taught in the Appraisal Institute's sales comparison approach courses, and this analysis is used to support adjustments in demonstration reports required for designation. However, there may be significant shortcomings in the paired sales technique to quantify adjustments. The other data analysis techniques, mentioned above, to quantify adjustments (grouped data analysis and secondary data analysis) also may have significant limitations. In addition, other available techniques, such as rank transformation regression (RTR) and price-quality regression (PQR), are missing from the list of data analysis techniques. RTR and PQR can be useful techniques with small sample sizes.

Purpose of the Study

The purpose of this study is to show how to predict the expected selling price of a property based on its productivity index. In this study, property productivity index regression (price-property productivity index regression) is applied in the valuation of a light industrial property. The weights, representing the relative importance placed on the productivity attributes by a buyer group, will be objectively determined given the data set selected. The property productivity index with value regression analysis is a useful technique with a small sample size.

In everyday practice, appraisers typically do not have at their disposal a large sample of comparable and competitive properties. Useful techniques to develop a reliable opinion of market value with small sample size include rank transformation regression and price-quality regression. In rank transformation regression, each comparable is ranked for each productivity attribute in a set of value-influencing attributes. In the second stage, an ordinary least squares regression analysis is developed, with selling price as the dependent variable and the ranking as the independent variable. In applying price-quality regression, each productivity attribute in a set of value-influencing attributes is rated for each comparable property. In the next step, subjective weights are assigned to each productivity attribute. Finally, a regression model is developed that has selling price as the dependent variable and the weighted property productivity index as the independent variable.

Literature Review

In the following discussion of the literature, small sample regression techniques are classified into two broad categories: rank transformation regression and price-quality regression. This classification scheme is for convenience and to facilitate exposition.

Rank Transformation Regression

In his textbook, *Appraisal Principles and Procedures*,[5] Babcock illustrates the use of ranking of property productivity attributes of single-family homes. In his technique, the physical description of each comparable property and of the subject property are placed on one side of cards, and on the back of the cards, the selling prices of the comparable sales are entered. The productivity attributes—size, age, condition, facilities, plan, etc., of each comparable property as well as the subject property are evaluated and then each is subjectively ranked, based on their productive capacities. In the next step, selling price is plot-

3. Ibid., 382.
4. Ibid., 398.
5. Henry A. Babcock, *Appraisal Principles and Procedures* (Homewood, IL: Richard D. Irwin, Inc., 1968).

ted against value rank and the value for the subject property is determined by the sale price corresponding to its value rank. In addition, mathematical calculation could be used to calculate the market value line (regression line). Babcock states that this is the technique used by personal property appraisers.[6]

In their *Appraisal Journal* article, Perry, Cronan, and Epley apply regression analysis to ranked data. The dependent variable is the ranked sale price and the independent variables are the rankings of the property productivity attributes for the comparable sales. They report that ranking data prior to applying ordinary least squares produces more accurate results than the conventional multiple regression analysis with small sample size.[7]

Price-Quality Regression

The genesis of price-quality regression can be traced to the "Methods of Mortgage Risk Rating" section of the 1936 underwriting manual of the Federal Housing Administration (FHA).[8] The FHA manual identifies twenty-eight risk features, which it classifies into four categories—the property, the location, the borrower, and the mortgage pattern. The total possible points for all the features in a category is 100 points. Each feature is assigned a weight. The weighted rating for each risk feature is derived by multiplying the rating of a feature by the weight for the feature.

In 1938, Babcock, Massey, and Greene present a location-rating scheme for residential property. In this early article they state,

> Valuation and risk rating should be carefully distinguished and placed in our minds in proper relation one to the other. Valuation is the determination of estimates of the values of properties. Risk rating is the process of thoroughly analyzing and measuring major factors of risk undertaken in the making of mortgage loans. It is apparent that risk rating is the broader of the two and that it includes valuation. The appraisal of property thus becomes a subordinate factor in the larger process.[9]

Later, Ratcliff offers criticism of the value-ranking method of valuation, stating, "This procedure incorporates the assumption that price varies directly with rank and assumes equal quality differences between ranks."[10] Keep in mind that ranked data is ordinal scale data and one cannot interpret differences between ranked values because the actual numbers used for ranking are arbitrary.

Ratcliff and Swan improve on the methodology of Babcock, Massey, and Greene by applying weights to the ratings of comparable properties for quality differentials. The weights applied to the productivity features of comparable properties are based on the judgment of the appraiser as to the feature's relative importance to the buyer group. For each comparable and the subject, each feature is rated in terms of relative quality and weighted by importance to the buyer group.[11]

Wendt in his textbook, *Real Estate Appraisal: Review and Outlook*, discusses Babcock's residential location-rating technique (a component of risk-rating procedure for residential mortgages) developed when he was with the Federal Housing Administration.[12] Wendt also cites Ratcliff and Swan for their illustrations of weighted feature rating technique in comparable analysis.[13] In addition, Wendt cites the weighted quality attributes technique applied in estimating capitalization rates.[14] Smith, in his textbook, presents a qualitative regression technique and applies it to

6. Ibid., 214–216.
7. Larry G. Perry, Timothy P. Cronan, and Donald R. Epley, "Ranking Comparable Properties Prior to Their Use in Regression on a Large or Small Sample," *The Appraisal Journal* (January 1986): 57–65.
8. Federal Housing Administration, *Underwriting Manual*, Rev. April 1, 1936 (Washington, DC): Part 1, 22–29 and Part II, 1–34.
9. Frederick M. Babcock, Maurice R. Massey, Jr., and Walter L. Greene, "Techniques of Residential Location Rating," *Journal of the American Institute of Real Estate Appraisers* (April 1938): 133–140.
10. Richard U. Ratcliff, *Valuation for Real Estate Decisions* (Santa Cruz, CA: Democrat Press, 1972): 154.
11. Richard U. Ratcliff and Dennis G. Swan, "Getting More from Comparables by Rating and Regression," *The Appraisal Journal* (January 1972): 68–75.
12. Paul F. Wendt, *Real Estate Appraisal: Review and Outlook* (Athens, GA: The University of Georgia Press, 1974): 75–77.
13. Ibid., 104.
14. Ibid., 32–133.

the valuation of an apartment property and one-unit residential property.[15] Graaskamp popularized the technique of price-quality regression with the publication of his monograph *The Appraisal of 25 N. Pinckney: A Demonstration Case for Contemporary Appraisal Method*.[16]

Clapp illustrates a technique to rate productivity attributes of properties. To rate a productivity attribute of the subject property and comparable properties, the highest measured productivity attribute is divided by lowest measured productivity attribute and the quotient is multiplied by 100. Doing it this way, the worst property on a productivity attribute is assigned a value of 100. The productivity attributes are then assigned weights, for example, based on discussions with real estate experts in the local market. Clapp calls the sum for a property its "amenity index."[17]

Recent authors applying price-quality regression include Wincott[18] and Rhodes.[19] Fanning illustrates the application of rating techniques to perform productivity analyses for different property types.[20]

The advantages of price-quality regression are (1) it is closer to a simulation of buyer behavior than the method of adjusting comparable sales for differences from subject property, (2) there are some productivity features that cannot be measured quantitatively, and (3) after adjusting the comparable properties, the appraiser through judgmental generalization or a weighted average of the adjusted sale prices would conclude an opinion of the defined value for the subject property. A weakness of the weighted rating technique is that both the ranking/rating and weights for property productivity attributes are subjectively determined by the analyst.

Research Design Methods and Procedures

As discussed in the preceding section, a weakness of the price-quality regression technique is that appraisers must rely on their own judgment of the preferences of buyers in order to determine the impact on price of the productivity features of properties and their ratings and weightings. In this study, the weights assigned to the productivity features are objectively determined through the application of the Solver program, a Microsoft Excel add-in (Microsoft Office: Home & Business 2016). The Solver program determines the weights for the different property productivity attributes so that the variance of the weighted scores for comparable properties is minimized. Minimizing the variance of the weighted productivity scores for the different properties produces an objective prediction of the market value for the subject property. The criterion of minimizing variance of the property productivity index is analogous to selecting comparable properties that are very close in their measured attributes. In the next step, a regression analysis is performed, using selling price as the dependent variable and the weighted productivity index or score as the independent variable.

Data Collection

In this case study, data are gathered on sales of single-tenant, light industrial property in the Omaha-Council Bluffs Metropolitan Statistical Area (MSA). The MSA is made up of five counties in southeastern Nebraska and three counties in southwestern Iowa. Data gathered on significant value- and rent-influencing variables is

15. Halbert C. Smith, *Real Estate Appraisal* (Columbus, OH: Grid, Inc., 1976): 132–136.

16. He applies the technique to a retail property. James A. Graaskamp, *The Appraisal of 25 N. Pinckney: A Demonstration Case for Contemporary Appraisal Methods* (Madison, WI: Landmark Research, Inc., 1977), 71–77.

17. John M. Clapp, *Handbook for Real Estate Market Analysis* (Englewood Cliffs, NJ: Prentice-Hall, Inc., 1987), 129–140.

18. D. Richard Wincott, "An Alternative Sales Analysis Approach for Vacant Land Valuation," *The Appraisal Journal* (Fall 2012): 310–317.

19. Gene Rhodes, "Qualitative Analyses in the Sales Comparison Approach Revisited," *The Appraisal Journal* (Fall 2014): 281–294.

20. Stephen F. Fanning, *Market Analysis for Real Estate: Concepts and Applications in Valuation and Highest and Best Use*, 2nd ed. (Chicago, IL: Appraisal Institute, 2014).

Exhibit 1 Summary of Sales

Sale	Building Type	Sale Price ($)	Date Sold	Building Size (SF)	Office Area (SF)	Percent Office (%)	Building Age (Yrs)	Ceiling Height (Ft)	Docks	Drive-In Doors	Docks plus Drive-In Doors	Sprinkler	Site Area	Sale Price ($) per Bldg SF
1	Warehouse	1,075,000	12/12/14	12,362	1,562	12.64	18	26.00	1	3	4	No	38,890	86.96
2	Warehouse	1,570,000	9/10/14	24,979	6,875	27.52	11	26.00	1	0	1	No	162,914	62.85
3	Warehouse	903,000	3/5/14	16,298	6,298	38.64	40	16.00	0	3	3	No	90,605	55.41
4	Warehouse	580,000	1/17/14	11,625	3,308	28.46	39	14.00	1	1	2	No	30,492	49.89
5	Warehouse	985,000	4/24/13	17,491	6,140	35.10	10	20.00	1	1	2	No	55,321	56.31
6	Warehouse	1,135,000	11/23/12	17,541	3,505	19.98	48	18.00	2	3	5	No	118,783	64.71
Subj.	Warehouse	1,100,000	10/2/14	16,455	3,672	22.32	25	20.00	1	3	4	No	78,844	66.85

based on a review of empirical real estate literature.[21] By basing the selection of property productivity features on empirical research, the subjective judgment of the analyst as to the value-influencing characteristics is minimized in the price-quality regression model. However, the subjective judgment of the analyst in selecting comparable properties remains. Exhibit 1 displays a summary of the gathered data.

Data Analysis

To analyze the gathered data, a regression analysis of the property productivity index and value model is developed. The dependent variable is selling price and the independent variable is the weighted property productivity index of the property productivity characteristics, as identified in the empirical real estate literature. The weighting of the property productivity characteristics is determined by applying the Solver program.

Rating Scale

In their analysis, Babcock, Massey, and Greene apply three different scales—namely, 1, 2, 3, 4, 5; 2, 4, 6, 8, 10; and 4, 8, 12, 16, 20—on a set of eight features for residential location.[22]

Ratcliff and Swan in their article state, "The extent of the scale required in rating the properties is determined by the degree of variation in the quality being rated." They apply a rating scale where for a property productivity feature the best property is rated 1 and other properties are rated in relation to the best property. In addition, they indicate that using only four numbers (1 to 4) may be adequate for minor degrees of difference.[23]

21. Brent W. Ambrose, "An Analysis of the Factors Affecting Light Industrial Property Valuation," *Journal of Real Estate Research* 5, no. 3 (Fall 1990): 355–370; Frank A. Fehribach, Ronald C. Rutherford, and Mark E. Eakin, "An Analysis of the Determinants of Industrial Property Valuation," *Journal of Real Estate Research* 8, no. 3 (Summer 1993): 365–376; William L. Atteberry and Ronald C. Rutherford, "Industrial Real Estate and Market Efficiency," *Journal of Real Estate Research* 8, no. 3 (Summer 1993): 376–399; Richard K. Buttimer, Jr., Ronald C. Rutherford, and Ron Witten, "Industrial Warehouse Rent Determinants in the Dallas/Fort Worth Area," *Journal of Real Estate Research* 13, no. 1 (1997): 47–55; and Bob Thompson and Sotiris Tsolacos, "Rent Adjustment and Forecasts in the Industrial Market," *Journal of Real Estate Research* 17, no. 1/2 (1999): 151–167.

22. Babcock, Massey, Jr., and Greene, "Techniques of Residential Location Rating."

23. Ratcliff and Swan, "Getting More from Comparables," 72. They apply the rating scale to the analysis of a one-unit dwelling.

Smith applies a scale of 1 to 5 (1 as the highest rating) to the appraisal of an eight-unit apartment and a one-unit residential property.[24] Similarly, Graaskamp employs a rating scale of 1, 3, 5, with a rating of 5 representing better than average and 1 reflecting worse than average. He states,

> We advise avoiding more elaborate scalers from 1 to 10 because the appraiser would have difficulty explaining small differences, for example, between 7 and 8, to either a client or a jury. It is easier to obtain agreement on the better than/worse than classification.[25]

Wincott applies a ten-point scale (1 to 10) in his analysis of vacant land. He indicates that "if the properties are fairly similar with respect to a particular attribute, only a portion of the range, say 4 to 7, may be sufficient to reflect the minor degrees of difference."[26]

Rhodes in his article uses five quality ratings: excellent (50), good (40), average (30), fair (20), and poor (10). He notes that "any range of numbers can be used as long as the spread between the numbers is the same, for example 25, 20, 15, 10, 5, or 100, 80, 60, 40, 20, or 5, 4, 3, 2, 1."[27]

Preston and Colman report, "On several indices of reliability, validity, and discriminating power, the two-point, three-point, and four-point scales performed relatively poorly, and indices were significantly higher for scales with more response categories, up to about 7."[28] Garland, in his review of journal articles on optimal number of categories, reports that many authors have concluded "the optimal number of scale categories is content specific and a function of the conditions of measurement."[29]

Ratings of Property Productivity Features

In the current article, the productivity features have been selected based on reports in the appraisal literature as to factors influencing value (see the data collection discussion). The ratings are based on appraisal and economic principles and logic. Graphic analysis is performed on the selected property productivity features to illustrate the relationship between the features and selling price.

Market Conditions (Date Sold)

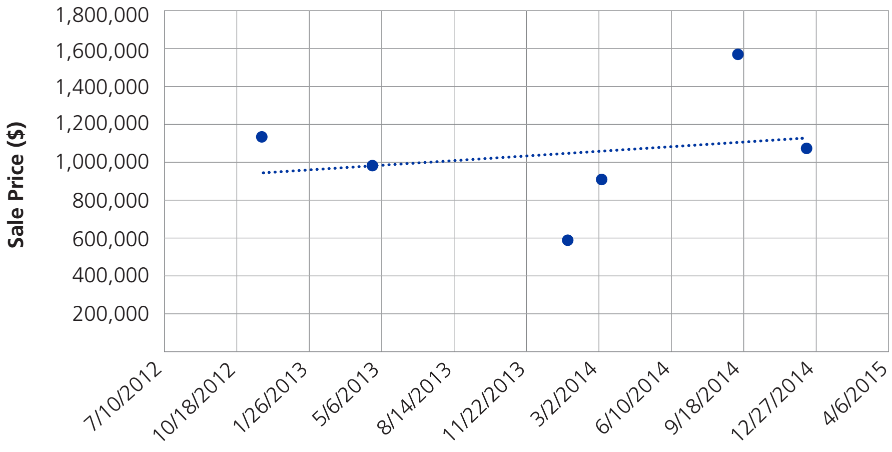
The adjustment for changes in market conditions is made to account for inflation, deflation, and changes in supply and demand between the date a comparable property sold and the effective date of the appraisal. If there are significant changes in market conditions, a comparable property would have sold for a different price as of the effective date of appraisal of the subject property. Therefore, an adjustment would be necessary for significant changes in market conditions.

In Exhibit 2, the graph shows a trend line with an upward trend in sale prices during the analysis period. The ratings table shows how the properties are rated on the market conditions attribute based on the date of sale.

24. Smith, *Real Estate Appraisal*, 132–136.
25. Graaskamp, *The Appraisal of 25 N. Pickney*, 71–72. The property type analyzed is a commercial property.
26. Wincott, "An Alternative Sales Analysis Approach," 313.
27. Rhodes, "Qualitative Analyses in the Sales Comparison Approach Revisited," 284. His analysis is related to vacant land.
28. Carol C. Preston and Andrew M. Colman, "Optimal Number of Response Categories in Rating Scales: Reliability, Validity, Discriminating Power, and Respondent Preferences," *Aceta Psychologica* 104, no. 1 (2000): 1–15.
29. Ron Garland, "The Mid-Point on a Rating Scale: Is it Desirable?" *Marketing Bulletin* 2 (1991): 66–70, Research Note 3.

Exhibit 2 Sale Price vs. Date Sold

Graphic Analysis of Market Conditions

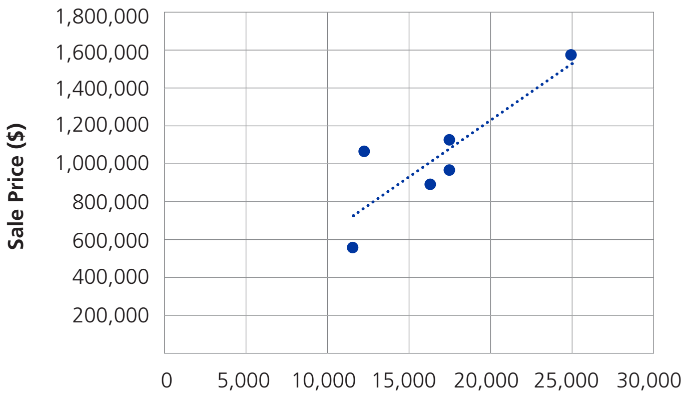


Market Condition Ratings

Date Sold	Rating
5/14/2011 – 5/13/2012	1
5/14/2012 – 5/13/2013	2
5/14/2013 – 5/13/2014	3
5/14/2014 – 5/13/2015	4
5/14/2015 – 5/13/2016	5

Exhibit 3 Sale Price vs. Building Size (SF)

Graphic Analysis of Building Size



Building Size Ratings

Building Size (SF)	Rating
< 5,000	1
5,000 – 9,999	2
10,000 – 14,999	3
15,000 – 19,999	4
20,000 – 24,999	5

Building Size

In Exhibit 3, the graphic analysis shows a trend line indicating that as the building size increases, the sale price also increases. Therefore, the larger the building, the higher the rating.[30] The building size ratings table shows how the properties are rated on the building size feature.

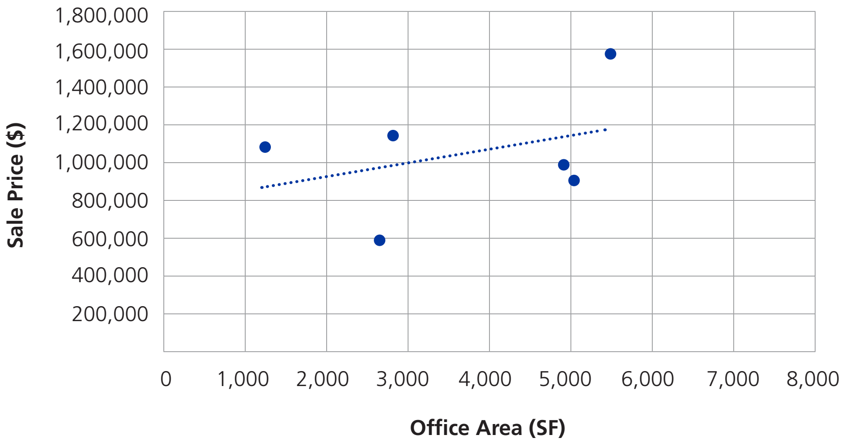
Office Area

In Exhibit 4, the graphic analysis shows a trend line indicating a positive relationship between sale price and the size of the office area. In terms of the cost to build-out the office area, the larger the office area, the higher the build-out cost. It should be noted that higher cost does not neces-

30. A plot of building size vs. price per square foot shows the typical inverse relationship between building size and price per square foot. The unit sale price decreases as the building size increases. In that case, a smaller building would be rated higher than a larger building, if analysis is based on price per square foot.

Exhibit 4 Sale Price vs. Office Area (SF)

Graphic Analysis of Office Area

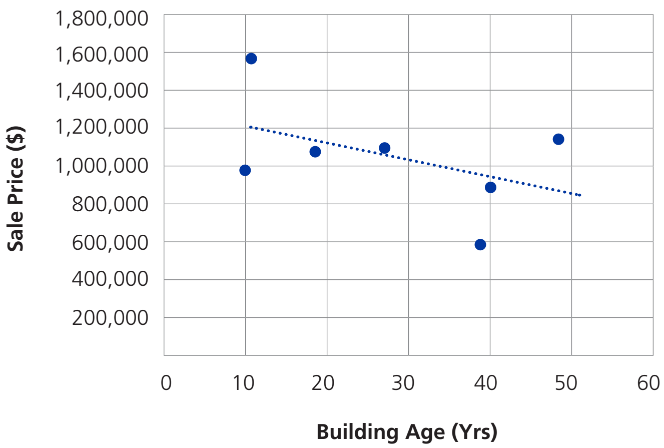


Office Area Ratings

Office Area (SF)	Rating
1,500 – 2,499	1
2,500 – 3,499	2
3,500 – 4,499	3
4,500 – 5,499	4
≥ 5,500	5

Exhibit 5 Sale Price vs. Building Age (Yrs)

Graphic Analysis of Building Age



Building Age Ratings

Building Age (Yrs)	Rating
≥ 50	1
40 – 49	2
30 – 39	3
20 – 29	4
< 20	5

sary translate to higher property value. For purposes of this article, the larger the office build-out area, the higher the rating, as shown in the table in Exhibit 4.

Building Age

The graphic analysis in Exhibit 5 reveals the expected negative relationship between sale price and building age. Ratings for building age also are presented, with ratings decreasing with age.

Ceiling Height

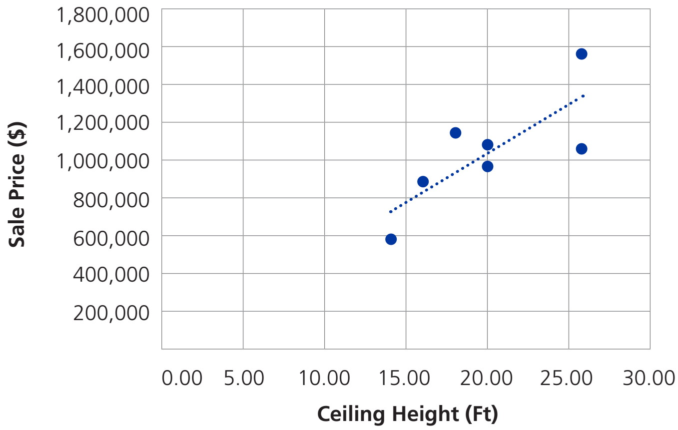
The graphic analysis in Exhibit 6 illustrates that the higher the ceiling height of a warehouse building, the higher the sale price. The rating table in Exhibit 6 assigns a higher rating to properties based on higher ceiling height.

Docks plus Drive-In Doors

The graphic analysis in Exhibit 7 shows there is a positive relationship between the total number of docks and drive-in doors of a property and the

Exhibit 6 Sale Price vs. Ceiling Height (Ft)

Graphic Analysis of Ceiling Height

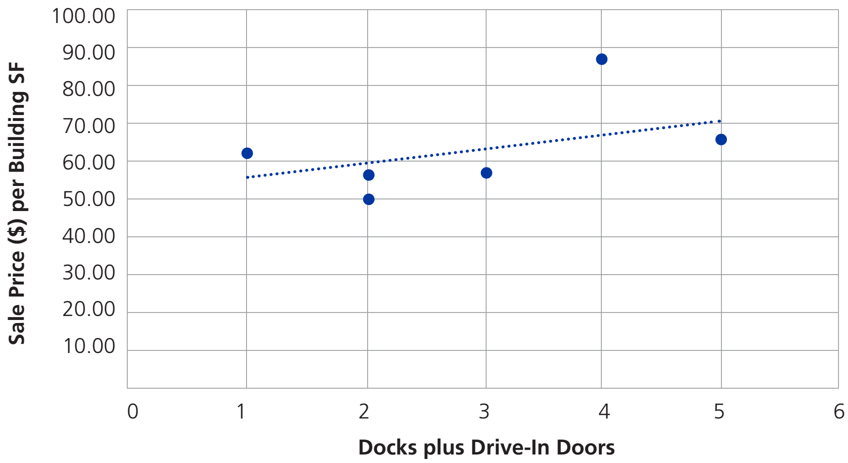


Ceiling Height Ratings

Ceiling Height (Ft)	Rating
< 10	1
10 – 14.99	2
15 – 19.99	3
20 – 24.99	4
≥ 25	5

Exhibit 7 Docks plus Drive-In Doors vs. Price per Square Foot

Graphic Analysis of Docks plus Drive-In Doors



Docks plus Drive-In Doors Ratings

Docks plus Drive-In Doors	Rating
1	1
2	2
3	3
4	4
5	5

price per square footage of warehouse building area. The table in Exhibit 7 presents the ratings related to these features.

Site Area

The graphic analysis in Exhibit 8 shows a positive relationship between sale price and site area along with the ratings for site area square footage.

Summary of Ratings

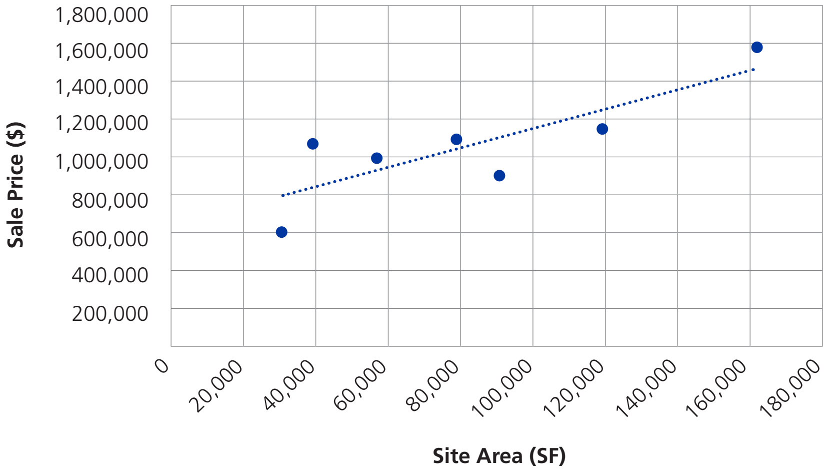
Based on the preceding discussion of ratings of the property productivity features, Exhibit 9 shows the ratings for each comparable property and the property appraised.

Application of the Solver Program

The Solver program is used to solve optimization problems. The Solver algorithm changes the values in the decision variables cells, subject to the limits in the constraint cells, and produces the desired result for the objective function cell. An optimization problem may be described as the problem of finding the best solution from all feasible solutions. In this instance, the optimization problem is minimizing the variance of the weighted property productivity indexes of the subject and comparable properties. The smaller this variance, the better the model would be for

Exhibit 8 Sale Price vs. Site Area (SF)

Graphic Analysis of Site Area



Site Area Ratings

Site Area (SF)	Rating
≥ 130,000	1
105,000 – 129,999	2
80,000 – 104,999	3
55,000 – 79,999	4
< 54,999	5

Exhibit 9 Property Productivity Features' Ratings

		Comparables					
Property Productivity Feature	**Subject**	1	2	3	4	5	6
Date Sold	4	4	4	3	3	2	2
Building Size	4	3	5	4	3	4	4
Office Area	3	1	5	5	2	5	3
Building Age	4	5	5	2	3	5	2
Ceiling Height	4	5	5	3	2	4	3
Docks plus Drive-In Doors	4	4	1	3	2	2	5
Site Area	2	1	5	3	1	2	4

predicting the selling price of the subject property. Note that the ordinary least squares estimator is an example of a technique to solve an optimization problem. The ordinary least squares estimator generates the set of values of coefficients of the predictor variables that minimize the sum of squared residuals. The residuals ($y - \hat{y}$) are calculated by subtracting the predicted value of the dependent variable ($\hat{y}$) from its actual value (y).

The mathematical model representing the optimization problem consists of the following three components:

1. Decision variables. Decision variables are the weights for the property productivity features of the subject and the comparable properties.
2. Objective. The objective is a mathematical expression in the decision variables. In this instance, the objective is to minimize the variance of the weighted property productivity indexes by selecting the weights for the property productivity features of the subject and comparable properties that achieve the objective.
3. Constraints. Constraints are the limitations or requirements of the problem, expressed as inequalities or equations in the decision variables. In this instance, the constraints are that the weights of the property productivity features are less than or equal to 100%. For example, for each property pro-

Exhibit 10 Weighted Property Productivity Index Setup

	A	B	C	D	E	F	G	H	I	J
3		Ratings								
4			Comparables							
5	**Property Productivity Feature**	**Subject**	1	2	3	4	5	6	**Weight**	**Variance**
6	Date Sold	4	4	4	3	3	2	2	100.00%	
7	Building Size	4	3	5	4	3	4	4	100.00%	
8	Office Area	3	1	4	5	4	5	2	100.00%	
9	Building Age	4	5	5	2	3	5	2	100.00%	
10	Ceiling Height	4	5	5	3	2	4	3	100.00%	
11	Docks plus Drive-In Doors	2	1	5	2	2	3	1	100.00%	
12	Site Area	2	1	5	3	1	1	4	100.00%	
13	Weighted Property Productivity Index	23.00	20.00	33.00	22.00	18.00	24.00	18.00		26.6190

ductivity feature of the subject property, the constraint is that the weight of its rating is less than or equal to 100%.

Model Components

Decision Variables. The initial setup of the analysis matrix is presented in Exhibit 10. In Exhibit 10, the decision variables for the subject property or a comparable property are the weights applied to the ratings of date sold, building area square feet, office area square feet, building age, ceiling height, docks plus drive-in doors, and site area square feet. The decision variables are contained in cells I6:I12 (i.e., I6, I7, I8, I9, I10, I11, I12). Cells B13:H13 (i.e., B13, C13, D13, E13, F13, G13, H13) contain the weighted property productivity indexes for the subject and comparable properties.

Before applying the Solver program, the weighted property productivity indexes are the weights assuming each property productivity feature receives a weight of 100%. The formulas for computing the weighted property productivity indexes for the subject and comparable properties are as follows:

Subject: (B6*I6) + (B7*I7) + (B8*I8) + (B9*I9) + (B10*I10) + (B11*I11) + (B12*I12)

Comparable 1: (C6*I6) + (C7*I7) + (C8*I8) + (C9*I9) + (C10*I10) + (C11*I11) + (C12*I12)

Comparable 2: (D6*I6) + (D7*I7) + (D8*I8) + (D9*I9) + (D10*I10) + (D11*I11) + (D12*I12)

Comparable 3: (E6*I6) + (E7*I7) + (E8*I8) + (E9*I9) + (E10*I10) + (E11*I11) + (E12*I12)

Comparable 4: (F6*I6) + (F7*I7) + (F8*I8) + (F9*I9) + (F10*I10) + (F11*I11) + (F12*I12)

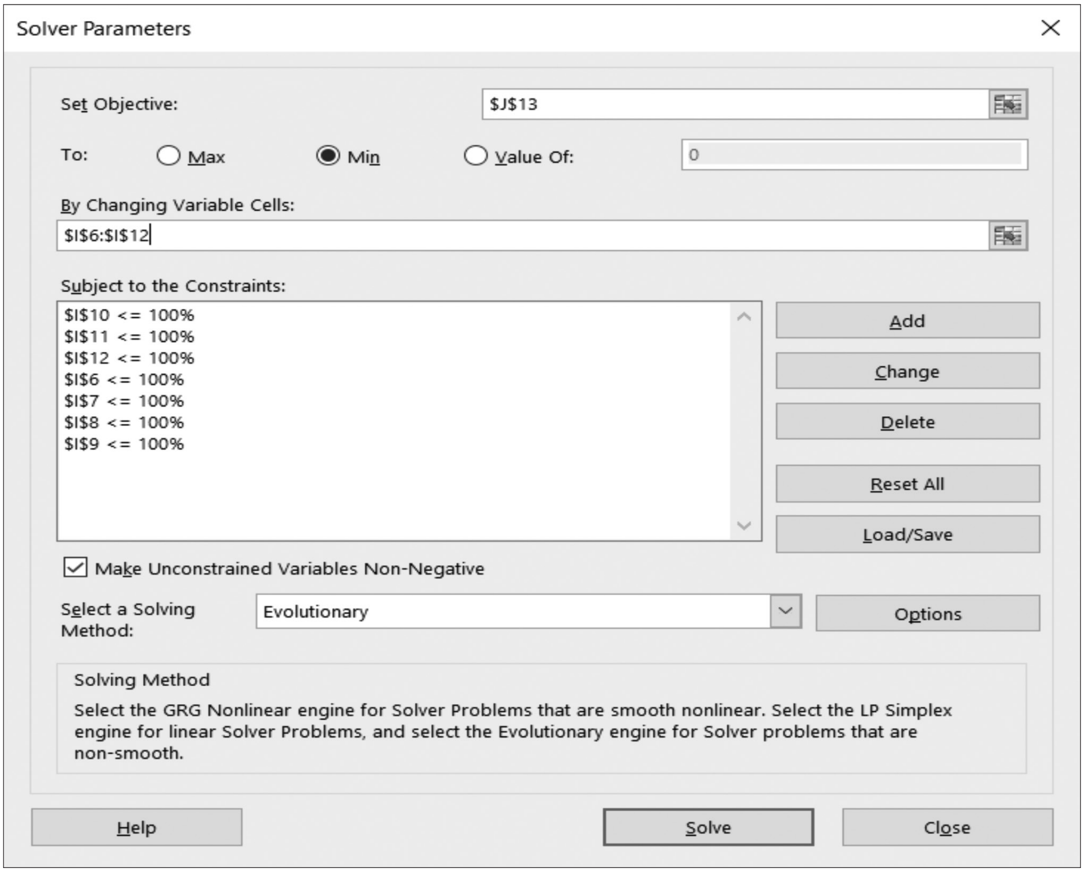
Comparable 5: (G6*I6) + (G7*I7) + (G8*I8) + (G9*I9) + (G10*I10) + (G11*I11) + (G12*I12)

Comparable 6: (H6*I6) + (H7*I7) + (H8*I8) + (H9*I9) + (H10*I10) + (H11*I11) + (H12*I12)

Objective Function. The objective is to minimize the variance of the weighted property productivity indexes for the subject and comparable properties, which are contained in cells B13:H13 (B13, C13, D13, E13, F13, G13, H13). The variance formula is contained in cell J13.

Constraints. In Exhibit 10, the entries for weight is 100% for each property productivity feature. In Exhibit 11, each constraint is that the weight for each property productivity feature is less than or equal to 100% for a property productivity feature for the subject and comparable properties. The mathematical formulas for constraints in Exhibit 11 are presented below:

Exhibit 11 Definitions of Solver Parameters



I6 <= 100%
I7 <= 100%
I8 <= 100%
I9 <= 100%
I10 <= 100%
I11 <= 100%
I12 <= 100%

Solving Method

Three algorithms options are available in the Solver program, namely, the Simplex method, the Generalized Reduced Gradient (GRD) Method, and the Evolutionary algorithm. The Simplex method assumes that the relationship between the objective function and constraints are linear functions of the decision variables. The GRD method assumes that the objective function and constraints are smooth nonlinear functions of the decision variables. The Evolutionary algorithm makes almost no assumptions about the relationships between the objective function and constraints and decision variables. The Evolutionary algorithm could be applied to both linear and nonlinear problems. In describing how the Evolutionary algorithm works, Dan Fylstra of Frontline Systems, states as follows:

> First, it relies in part on random sampling. This makes it a nondeterministic method, which may yield different solutions on different runs—even if the user hasn't changed the model at all. In contrast, the Simplex and GRG Solvers are deterministic methods—they always yield the same solution for given starting values of the decision variables.[31]

31. Dan Fylstra, "Backgrounder—Genetic and Evolutionary Algorithms versus Classical Optimization," Frontline Systems, http://www.solver.com/press/backgrounder-genetic-and-evolutionary-algorithms-versus-classical-optimization (accessed February 27, 2016).

Exhibit 12 Ratings and Weighted Property Productivity Indexes

	A	B	C	D	E	F	G	H	I	J
3		Ratings								
4			Comparables							
5	**Property Productivity Feature**	**Subject**	1	2	3	4	5	6	**Weight**	**Variance**
6	Date Sold	4	4	4	3	3	2	2	0.45%	
7	Building Size	4	3	5	4	3	4	4	0.57%	
8	Office Area	3	1	4	5	4	5	2	0.08%	
9	Building Age	4	5	5	2	3	5	2	0.14%	
10	Ceiling Height	4	5	5	3	2	4	3	0.21%	
11	Docks plus Drive-In Doors	2	1	5	2	2	3	1	0.13%	
12	Site Area	2	1	5	3	1	1	4	0.23%	
13	Weighted Property Productivity Index	0.0643	0.0568	0.0851	0.0589	0.0470	0.0572	0.0530		0.000147

The Evolutionary algorithm is used for this analysis. Exhibit 10 shows the setup before the Solver program is used to estimate the weights for the property features using the Evolutionary algorithm. The weights are selected to minimize the variance of the weighted index. As stated before, the criterion of minimizing variance of the property productivity index is analogous to selecting comparable properties that are very close in their measured attributes. Exhibit 11 shows the definitions of parameters for the Solver program.

First Run of the Solver Program

Exhibit 12 presents the weights for the various property productivity features and the weighted property productivity indexes for the comparable and subject properties after running the Solver program. Next, the weighted property productivity indexes presented in Exhibit 12 are applied in a simple regression to predict the expected selling price of the subject property. Exhibit 13 presents the data set for the simple regression analysis. In next step, Excel is used for the regression analysis. Regression is one of the analysis tools in the Data Analysis add-in of Excel. Exhibit 14 presents the simple regression output.

Predicting Expected Selling Price of Property Appraised

The r-square for the estimated model is 66.00%, and this indicates the percent variation in the selling price that is explained by the weighted property productivity indexes for the comparable properties. Using the coefficients for the intercept term and weighted property productivity index extracted from Exhibit 14, the estimated equation follows:

$$\ln(\text{Selling Price}) = 12.61237897 + (20.1309541 * \text{Weighted Property Productivity Index})$$

To predict the expected selling price of the subject property, its weighted property productivity index (0.064276925) is substituted into the above equation, as follows:

$$\ln(\text{Selling Price}) = 12.61237897 + (20.1309541 * 0.064276925) = \$13.90633479$$

The predicted selling price is in natural log unit. Therefore, to convert it to its original unit, we would find its antilog or its exponent to base *e*. The antilog of $13.90633479 is $1,095,076.51. The property appraised sold on 10/2/2014 for $1,100,000. The difference between the sale price and the predicted selling price ($1,095,077) is 0.45%.

Exhibit 13 Simple Regression Data Set

Sale	Sale Price ($)	ln(Sale Price) ($)	Weighted Property Productivity Index
1	1,075,000	13.887831	0.056804060
2	1,570,000	14.266586	0.085110728
3	903,000	13.713478	0.058850697
4	580,000	13.270783	0.047009373
5	985,000	13.800397	0.057196635
6	1,135,000	13.942143	0.053031655
Subject	1,100,000	13.910821	0.064276925

Exhibit 14 Summary Regression Output

Regression Statistics	
Multiple R	0.812425924
R Square	0.660035882
Adjusted R Square	0.575044852
Standard Error	0.21265446
Observations	6

ANOVA

	df	SS	MS	F	Significance F
Regression	1	0.351191056	0.351191056	7.765947594	0.049476245
Residual	4	0.180887677	0.045221919		
Total	5	0.532078733			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	12.61237897	0.439681435	28.68526608	8.79033E-06	11.3916276	13.83313033
Weighted Property Productivity Index	20.1309541	7.223823577	2.786744982	0.049476245	0.074404492	40.18750372

Exhibit 15 Random Sampling of Weights for Property Productivity Index Calculation and Predicted Selling Price for Subject Property

Solver Program Run	Weighted Property Productivity Index—Comparable Properties and Subject Property							Predicted Selling Price ($) of Subject Property
	1	2	3	4	5	6	Subject	
1	0.05680406	0.085110728	0.058850697	0.047009373	0.057196635	0.053031655	0.064276925	1,095,076.51
2	0.095813751	0.149635121	0.11754515	0.091275685	0.114164741	0.093288539	0.114790	1,042,760.17
3	0.182824248	0.271518549	0.161700	0.132937853	0.18486506	0.156364107	0.185427017	1,019,579.78
4	0.047785254	0.068458331	0.045747091	0.036843685	0.040508176	0.039785224	0.052150	1,130,429.85
5	0.066627774	0.07640463	0.050369132	0.052620719	0.062548068	0.035744024	0.065456431	1,078,472.23
6	0.001194159	0.001596233	0.001173543	0.000962696	0.001240	0.000945622	0.001275664	1,091,809.72
7	0.036662404	0.046666217	0.03173354	0.032367354	0.044298603	0.021041988	0.037461774	1,022,316.39
8	0.041647356	0.07747602	0.06016211	0.04805702	0.056545039	0.047410817	0.05692921	1,022,236.27
9	0.06205073	0.100258392	0.068477833	0.061719167	0.075635785	0.047658905	0.071993015	1,023,592.54
10	0.016078337	0.025676694	0.019157833	0.014963458	0.016492536	0.01458064	0.018898412	1,054,350.02
11	0.005600	0.008585304	0.007391966	0.006275704	0.006699384	0.005154387	0.007243691	1,058,860.45
12	0.007387912	0.013605741	0.009519205	0.006735889	0.007271452	0.008552877	0.009190	1,031,760.46
13	0.031825585	0.063881941	0.045200606	0.031517752	0.046086978	0.036054399	0.039791402	951,281.97
14	0.0611084	0.09930312	0.051848208	0.043783123	0.068508537	0.047382735	0.062695893	1,006,793.45
15	0.022851452	0.037790	0.023121975	0.018700839	0.02472453	0.023686442	0.025870	1,029,276.17
16	0.061753373	0.09657662	0.055008116	0.0485708	0.065708054	0.053568295	0.066132916	1,037,873.73
17	0.054245503	0.076306977	0.058238536	0.043015827	0.057850	0.047263756	0.058880	1,059,864.79
18	0.087111698	0.123841779	0.081801004	0.070105453	0.101801684	0.073231984	0.091894906	1,025,278.67
19	0.04742282	0.06625054	0.048532548	0.043553577	0.06450573	0.035963117	0.051508248	1,003,830.40
20	0.027954626	0.042087653	0.029828745	0.023801259	0.028036378	0.024192746	0.031245426	1,069,205.64
21	0.031150	0.05835636	0.047168452	0.039527908	0.051425682	0.031300611	0.041432094	981,547.96
22	0.066759093	0.119082775	0.089210	0.069803734	0.076645795	0.072857235	0.085764262	1,034,543.29
23	0.055693925	0.085200	0.071122488	0.057947433	0.081868006	0.045227359	0.065976251	996,602.89
24	0.045901215	0.070726793	0.049014762	0.040764054	0.042354794	0.041939067	0.053564466	1,109,287.05
25	0.055760	0.081180	0.055744549	0.041177761	0.053370	0.055933256	0.061072897	1,090,582.94
26	0.06087292	0.080033234	0.059302043	0.05067942	0.063210	0.042633056	0.065054115	1,085,469.18
27	0.020843536	0.029647648	0.020190714	0.013472639	0.020930	0.021148396	0.021802756	1,045,834.51
28	0.067776454	0.11219772	0.099031456	0.085685881	0.106608583	0.065106793	0.091060	1,003,412.35
29	0.066757521	0.085843456	0.062366764	0.054273128	0.068140	0.048511248	0.070156433	1,096,423.36

Additional Runs of the Solver Program

As previously discussed, the Evolutionary algorithm is selected to generate the weighted property productivity indexes for the properties. The Evolutionary algorithm uses random sampling; therefore, different results may be generated with multiple runs of the program. Exhibit 15 presents the results of the additional runs of the Solver program.

Statistical Analysis of Predicted Selling Prices

Exhibit 15 shows the results for 29 runs of the Solver program. Next, the predicted selling prices are subjected to statistical analysis in order to conclude an expected selling price for the subject property with a 95% prediction confidence interval. Exhibit 16 shows the output of descriptive analysis using Excel.

Exhibit 16 Statistical Analysis Results

Mean ($)	**1,044,770.78**
Standard Error ($)	**7,676.12**
Median ($)	1,037,873.73
Mode	N/A
Standard Deviation ($)	**41,337.16**
Kurtosis	-0.236799739
Skewness	0.027752944
Range ($)	179,147.88
Minimum ($)	951,281.97
Maximum ($)	1,130,429.85
Count	29
Confidence Level (95.0%) ($)	**15,723.81**

Confidence Interval for Mean of Predicted Selling Price[32]

The formula for calculating the confidence interval, $100(1–\alpha)\%$, for the mean when the population standard deviation σ is not known is

$$x\text{-}bar \pm t_{\alpha/2df}(s/\sqrt{n})$$

where:

$x\text{-}bar$ = sample mean

$t_{\alpha/2df}$ = t_{df} value corresponding to the probability of $\alpha/2$ in the upper-tail of the distribution with $df = n - 1$

s = standard deviation of the sample

n = sample size

$s/\sqrt{n}$ = standard error of the mean

The 95% confidence interval (α = 5%) for the mean predicted selling price is computed as

$$x\text{-}bar \pm t_{\alpha/2df}(s/\sqrt{n})$$

$$= \$1{,}044{,}770.78 \pm 2.048(\$41{,}337.16/\sqrt{29})$$

$$= \$1{,}044{,}770.78 \pm 2.048(\$41{,}337.16/5.39)$$

$$= \$1{,}044{,}770.78 \pm 2.048(\$7{,}669.23)$$

$$= \$1{,}044{,}770.78 \pm \$15{,}706.59$$

Therefore, one would conclude, with 95% level of confidence, that the expected selling price for the subject property is between \$1,029,064.19 and \$1,060,477.37. Detailed notes for this computation are shown below.

$s/\sqrt{n}$ = (\$41,337.16/$\sqrt{29}$) = \$7,669.23. Except for rounding error, it is the same as that shown (standard error \$7,676.12) in Exhibit 16.

$t_{\alpha/2df}$ = t value at $\alpha/2$ (5%/2) or 0.025 at $n - 1$ degrees of freedom (df) or 29 – 1 (28) = 2.048.

$t_{\alpha/2df}(s/\sqrt{n})$ = 2.048(\$7,669.23) = \$15,706.23. Except for rounding error, it is the same as that indicated (\$15,723.81) for a confidence level (95%) in Exhibit 16.

32. This discussion is based on Kenneth M. Lusht, *Real Estate Valuation, Principles and Applications* (State College, PA: KML Publishing, 2001), 119–138.

Conclusion

In everyday appraisal practice, the analyst is faced with a limited sample size available for analysis. Applying the price-quality regression technique has been advocated in the real estate literature as a way to solve the limited sample size problem. The criticisms of the price-quality regression technique are that property productivity features of a property, the ranking/rating of productivity features, and the weights assigned to the productivity features are subjectively determined by the analyst.

In this study, two of the above-referenced criticisms of the price-quality regression are resolved by relying on the empirical literature for the selection of significant property productivity features of a property class and assigning them weights by applying Solver program. Rating the property productivity features of properties mirrors the price-establishing behavior of the relevant buyer group in purchase decision making. A limitation of the study is that some significant property productivity characteristics may have been omitted from the model. A second limitation is that the analyst will still subjectively rank/rate the productivity features for the subject property and the set of comparable properties selected for analysis.

About the Author

A. Ason Okoruwa, PhD, MAI, AI-GRS, is the president of Bedrock Valuation and Consulting Services Corp. in Omaha, Nebraska. He has been engaged in valuation of real property, consulting, market analysis, and litigation support, since 1996. He previously taught at North Carolina Central University and the University of Northern Iowa. He has published in *The Appraisal Journal*, *Journal of Real Estate Research*, *Journal of Property Research*, *Journal of Urban Economics*, *Communications in Statistics*, *European Journal of Operational Research*, and *Journal of Information and Optimization Sciences*. He received his BBA in health administration and MBA in finance from Georgia State University, Atlanta, Georgia, and PhD in business, with a concentration in real estate, from the University of Georgia, Athens, Georgia.
Contact: ason@bedrockvacs.com

Additional Reading

Suggested by the Author

Colwell, Peter F., John A. Heller, and Joseph W. Trefzger. "Expert Testimony: Regression Analysis and Other Systematic Methodologies." *The Appraisal Journal* (Summer 2009): 253–262.

Conover, W. J., and Ronald L. Iman. "Rank Transformations as a Bridge Between Parametric and Nonparametric Statistics." *The American Statistician* 35, no. 3 (1981): 124–129.

Grissom, Terry V., Rudy R. Robinson, and Ko Wang. "A Matched Pairs Analysis Program in Compliance with FHLBB Memorandum R 41B/C." *The Appraisal Journal* (January 1987): 42–68.

Iman, Ronald L., and W. J. Conover. "The Use of the Rank Transform in Regression." *Technometrics* 21, no. 4 (1979): 499–509.

Wolverton, Marvin L. "Empirical Investigation into the Limitations of the Normative Paired Sales Adjustment Method." *Journal of Real Estate Research* 15, no. 1/2 (1998): 191–203.

Additional Resources

Resources suggested by the Y. T. and Louise Lee Lum Library

Appraisal Institute

- **Education**
  http://www.appraisalinstitute.org/assets/1/7/aiedcat.pdf
  - *Advanced Spreadsheet Modeling for Valuation Applications*
  - *Application & Interpretation of Simple Linear Regression*
  - *Commercial Real Estate Training: Appraisal Engagement, Appraisal Review and Evaluation*
  - *Comparative Analysis*
  - *General Appraiser Sales Comparison Approach*
  - *Quantitative Analysis*
  - *Residential Sales Comparison and Income Approach*
  - *Residential Applications: Using Technology to Measure and Support Assignment Results*
  - *Residential Applications Part 2: Using Microsoft Excel to Analyze and Support Appraisal Assignment Results*

- **Publications**
  - *An Introduction to Statistics for Appraisers*
    http://www.appraisalinstitute.org/an-introduction-to-statistics-for-appraisers/
  - *Valuation by Comparison: Residential Analysis and Logic*
    http://www.appraisalinstitute.org/valuation-by-comparison-residential-analysis-and-logic/